Spring 2010

Rock, Paper, Scissors Shoot: an Exploration of the Refinements of the Nash Equilibrium and their Applications to the Housing Market Collapse

Maureen McDaniel
Regis University

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ROCK, PAPER, SCISSORS SHOOT: AN EXPLORATION OF THE REFINEMENTS OF THE NASH EQUILIBRIUM AND THEIR APPLICATIONS TO THE HOUSING MARKET COLLAPSE

A thesis submitted to
Regis College
The Honors Program
in partial fulfillment of the requirements
for Graduation with Honors

by

Maureen McDaniel

May 2010

Thesis written by

Maureen McDaniel

Approved by

_____
Thesis Advisor

_____
Thesis Reader or Co-Advisor

Accepted by

_____
Director, University Honors Program

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Chapter One: Background

Just about everyone has played the game rock, paper, scissors. While this childhood game seems simple enough, there are layers of underlying theoretical complexity that transform this game into the epitome of game theory: "the study of the ways in which strategic interactions among rational players produce outcomes with respect to the preferences (or utilities) of those players." [1] This paper will explain the basic concepts of game theory, focusing on the subtleties and refinements of the Nash Equilibrium, a specific solution concept, and its application to the housing market collapse of 2007.

The Nash Equilibrium is named for John Nash who was born in Bluefield, West Virginia on June 13, 1928. Nash received his bachelor's of science and his master's degrees in mathematics from Carnegie Mellon University, previously Carnegie Tech., and received his doctorate from Princeton University. While at Carnegie Mellon, Nash took a class on International Economics which led him to write the paper "The Bargaining Problem." [2] This idea, coupled with the existing work by mathematicians John von Neumann and Oskar Morgenstern, led to his interest in game theory while at Princeton. [3] At age 21, Nash wrote his dissertation on game theory, discovering a

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[1] Ross, Don. "Game Theory." *Stanford Encyclopedia of Philosophy*. 3/10/06. http://plato.stanford.edu/entries/game-theory/
[2] Nash, J.F. (1950). "The Bargaining Problem." *Econometrica* 18, 155-162.
[3] Nash, John. "Autobiography." *The Nobel Foundation*. 1994 NobelPrize.org

universal solution concept for all non-cooperative games[4] termed "The Nash Equilibrium." This prodigious discovery won Nash the Nobel Prize in Economics in 1994 when he was 66 years old.[5] Today the Nash Equilibrium is considered "maybe the most important solution concept in game theory," and is used extensively in economics.[6] A few economic examples that utilize game theory include bargaining, auctioning, the utilization and distribution of materials, and other scenarios in which two or more parties are in conflict. Consider the specific example of an entrepreneur deciding to enter a monopolistic market. He must make his decision whether or not to enter this market based not only his own reward but also the decisions of his competitor. If his competitor chooses to fight him, then the entrepreneur might actually lose more than he could gain by entering the market, thereby deterring him from doing so. The solution in a scenario where one person's decision is based on another's is an example of the Nash Equilibrium.

Game theory is based on the study of rational players interacting. The classical view of a rational player is someone who "maximizes his/her objective functions given his/her beliefs about the environment."[7] In essence, this means a player is continuously striving to achieve the best possible outcome for himself. An objective function, also known as an utility function in economics, quantitatively describes a rational decision maker's preferences for one outcome over another. The existence of the utility function is based upon the *expected utility maximization theorem* which claims that for every

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[4] Term to be explained in future chapter.
[5] "John F. Nash, Nobel Prize Winner in Econometric Sciences, at HEC Montreal." *HEC Montreal Headlines.* 5/16/05 http://www2.hec.ca/en/headlines/2005/2005015_en.html
[6] Myerson, Roger. *Game Theory Analysis of Conflict.* pg. 105. Harvard University Press, Mass. 1991.
[7] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 1. Academic Press, New York. 1993.

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possible outcome, it is possible to assign utility numbers to that outcome which enables the player to pick the outcome with the highest utility number maximizing his utility function. According to Roger Myerson, author of *Game Theory Analysis of Conflict*, "any rational decision maker's behavior should be describable by a utility function . . . and a subjective probability function which characterizes his beliefs about all relevant unknown factors."[8] Examples of relevant unknown factors include the decisions of the other players, the characteristics of those players, and their own personal preferences.

Some economists believe that a player's drive to maximize his utility function ultimately makes him selfish. This concept is debated, as there are many examples of individuals who choose to act selflessly. However, some theorize that selfless individuals simply place a greater value or utility on the happiness of others, so ultimately, that individual is still trying to maximize his own objective function. However, work by Nobel Prize Winner Elinor Ostrom and other economists in the field suggest that people are more prone to collective action, placing the needs of the community above their own. All sides of the debate must be considered when analyzing any economic situation. For example, in economics, utility is a person's preference for one outcome over another. By analyzing utility, game theory is trying to place numerical values on human preferences, which can prove to be a difficult task. This is, however, exactly what game theory strives to do: to model behavior so that it can be analyzed quantitatively rather than qualitatively.

In addition to the assumption of a rational player, game theory also assumes the player is *intelligent* which means "he knows everything that we [those educated in game

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[8] Myerson, Roger. *Game Theory Analysis of Conflict*. pg. 5. Harvard University Press, Mass. 1991.

theory] know about the game and he can make any inferences about the situation that we can make."[9] In essence, the subtleties of the various types of games are not lost on the player, and he can make educated decisions. An economic example in which the players are assumed rational but not intelligent is price theory. Experts in economic price theory assume the players are rational, that they will follow the rules of demand by buying the cheaper of products deemed identical. However, the experts do not assume that the consuming public understands the deeper underlying laws of supply and demand and the consequences these laws produce on the surrounding markets. This example shows that when certain assumptions regarding rational and intelligent players are not met, the scenario, even if it is an economic situation that deals with conflict, cannot be analyzed with game theory.

The assumption of intelligence has serious implications, especially when much in economics deals with the general public. First, we must understand that the underlying purpose of game theory is to analyze the game at hand and then use this information to predict the path of play for future players and scenarios. If analysts predict how a specific game will unfold, then businesses and public policy makers can use the resulting models to help determine the most probable outcome. However, if these models are based upon this assumption of intelligence, this means that the average human being who does not understand the rules of game theory might react differently than the model predicts. This would obviously cause problems for those relying on the accuracy of the model. So, when dealing with game theoretic analysis, it is important to remember the

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[9] Myerson, Roger. *Game Theory Analysis of Conflict*. pg. 4. Harvard University Press, Mass. 1991.

assumptions upon which the analysis is based. Outcomes may stray far from the predictions if the assumptions do not hold. Therefore, game theory has limited applicability.

The majority of the definitions necessary to describe game theory come from Eichberger's *Game Theory for Economists* (unless specifically stated otherwise). First, we need the tools to describe and analyze the games and their characteristics. A basic game can be represented in three main forms. The first and most explicit is called the *extensive form* in which all the necessary information about the game is shared. This includes the sequence of moves, most often illustrated by a game tree, also called a decision tree. For example, consider the game of rock, paper, scissors. The first player has the option of playing any of these three moves. This is illustrated by the original *node*, or possible game situation as seen by a dot in the game tree; the original node is called the *root node.*[10] The three branches represent each possible *action,* or move from node to node. Player two then also can choose three varying actions from the current existing node arrived at by player one. Obviously, while playing the real game, the players make their decisions simultaneously, and in most game-theoretic analyses, we assume simultaneous decision making. However, the game tree below appears as if the decisions are made sequentially, which we know is not the case for this particular game. Yet, the game tree allows us to see every possible combination of decisions that can be made by all players. Below is the game tree for the game rock, paper, scissors.

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[10] Myerson, Roger. *Game Theory Analysis of Conflict.* pg. 38. Harvard University Press, Mass. 1991.

Figure 1: Game Tree for one round of Rock, Paper, Scissors

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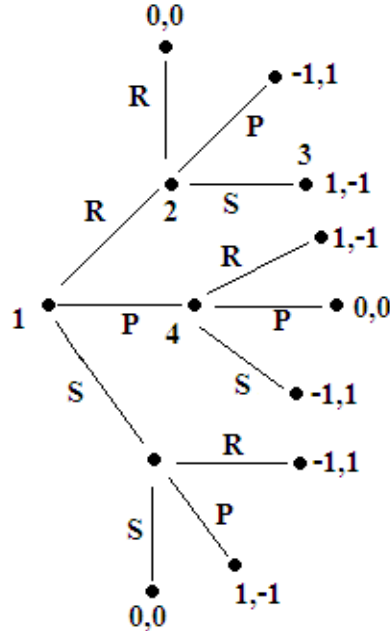
Not only does the game tree show the various sequences of possible actions and nodes, but it also shows the possible *payoff function* for each outcome or *terminal node,* the final node where the game ends and no further actions take place. Note that every node is either a terminal node, a *chance node,* or a *decision node.* A chance node is "where the next branch in the path of play would be determined by some random mechanism, according to the probabilities that are shown on the branches that follow the chance node."[11] There are no chance nodes in the game of rock paper scissors, but imagine a game where if a player draws a red card and then he has to draw another card, but if he draws a black card he does not. This is an example of a chance node because

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[11] Myerson, Roger. *Game Theory Analysis of Conflict.* pg. 39. Harvard University Press, Mass. 1991.

the next action in the path of play is decided by a random mechanism as opposed to a decision made by a player.

Decision nodes, on the other hand, represent decisions made by the players as to the next action in the path of player. For example, returning to the rock, paper, scissors example, each player decides which symbol to play, so each node in this game tree is a decision node, except for the final terminal nodes. Let it be clear that not all nodes illustrate decision opportunities for both players. Some nodes, deemed *decision nodes of a player* may represent a decision which can only be made by one specific player. An example of this can be seen on the game tree at node 1. This node is a decision node for player one only, while node 2 is a decision node for player two only. By differentiating between the two, one can analyze sequences of moves more accurately.

The *payoff function* associates each terminal node with a vector of real numbers representing the gains or losses assumed by each player at that terminal node. For example, if player one chooses to play "rock," which sends her from node 1 to node 2, and then player two chooses to play "scissors" which moves him from node 2 to terminal node 3, the appropriate payoff function is then (1, -1), where player one wins one point and player two loses one point. Through the use of a game tree, all the possible "stages of the interaction, the conditions under which the player has to move, the information a [player] holds at different stages, and the motivation of the [player]"[12] is apparent, which is required for a game in the extended form. The information a player holds at different stages of the game is called an *information set*, or the set of nodes that are

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[12] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 2 Academic Press, New York. 1993.

indistinguishable. For example, node 2 and node 4 are in the same information set

because player two would not be able to distinguish between the two nodes if he were

unaware of player one's previous move.

The next common form for representing games is the *strategic form* or *normal*

form which is far sparser that the original extended form. In the strategic form, only

possible strategies, the set of players, and payoff functions are provided.   The most

common way to present this information is with a payoff matrix.

Figure 2: Payoff Matrix for Rock, Paper, Scissors

		Player Two		
		Rock	Paper	Scissors
Player One	Rock	(0,0)	(-1,1)	(1,-1)
	Paper	(1,-1)	(0,0)	(-1,1)
	Scissors	(-1,1)	(1,-1)	(0,0)

In this form, much of the information regarding the sequence of moves is left out,

emphasizing a focus on strategy as opposed to the game's dynamic structure of sequential

moves. While the moves of rock, paper, scissors do happen simultaneously in real life,

the game tree from the extensive form shows there is still a sequence of moves that can

occur, and this information is left out of the strategic form. Because of this, the players

are forced to analyze only the information given to them, i.e., the payoff functions and

the appropriate strategies that will result in the most favorable payoff.

Within these two main games forms, there are many other types of games, each

with varying characteristics. First, there are *games with complete information* where

every aspect, including the number of players, the set of all nodes, the set of all actions,

all information sets, and all payoffs, is considered "common knowledge," meaning that

all of the players are fully informed of every aspect, and all the players are aware that the other players are also fully informed. This contrasts with a *game with incomplete information* where at least one element of the game is unknown. It is also possible for players to have *private information,* meaning some information is not common knowledge amongst all of the players.[13] There are also *games with perfect information* in which each player is informed of all previous moves made by all players. This is in contrast to *games with imperfect information* in which there is a level of uncertainty concerning previous moves. *Perfect* versus *imperfect information* is deemed a "structural property," or in other words a style of play. *Complete* versus *incomplete information* is an "informational characteristic" of the game displaying how much initial information is known by the players and has nothing to do with the style of play.

In order to fully describe game theory, we must also understand strategies. A strategy "is a complete plan for playing the game. In this context, 'complete' means that for any contingency the plan must specify what the player would do."[14] The first type of strategy is a *pure strategy* which describes a unique action that will be taken if a specific information set is reached. A *pure strategy combination* is a list of pure strategies for all of the individuals in the game at that specific information set. Now compare a pure strategy to a *mixed strategy* in which "a player chooses a random device for selecting which pure strategy to play."[15] Basically, the player is assigning a probability distribution to the pure strategy at a given information set so the player can make an

[13] Myerson, Roger. *Game Theory Analysis of Conflict.* Pg.64. Harvard University Press, Mass. 1991.
[14] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 17 Academic Press, New York. 1993.
[15] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 20 Academic Press, New York. 1993.

educated guess about which strategy to play depending on the probability. This is necessary when information regarding the previous player's moves is unavailable, in other words a game with imperfect information. If the player cannot distinguish his current node, he cannot choose which strategy would result in the best outcome, so he assigns a probability distribution to the pure strategies and uses that to make his decision. For example, in rock, paper, scissors, if player two cannot distinguish his current node, he could utilize a previously determined probability distribution assigned to the original actions of player one to help him determine his location. So, assume player one has historically played rock 65% of the time. This prompts player two to utilize this information to make an educated guess about his node location, and in turn play paper, as he is 65% certain that he will win with this strategy.

The third type of strategy is a *behavior strategy* in which the player randomly chooses which strategy to use at each information set. Unlike mixed strategies, a probability function is not assigned to the individual decision nodes but instead probabilities are assigned to the terminal nodes. Returning to the example of rock, paper, scissors, if no previously determined probability distribution existed as was the case in the mixed strategies example, he could instead guess his location randomly. This inherently places a uniform probability distribution at his final destination, or terminal node, as there will be a certain probability he will end there. For example, if no previous data is collected about player one's typical moves, player two guesses randomly that player one played scissors. This means, player two will play rock, which creates a resulting 1/3 probability that he is indeed at that resulting terminal node. Due to the lack

of historical data, the player switched from a utilization of mixed strategies to a uniform probability distribution placed upon the terminal nodes.

Understanding the Nash Equilibrium requires an understanding of cooperative versus noncooperative game theory. C*ooperative game theory* deals with groups of players who may share joint outcomes. *Noncooperative games theory* focuses instead on the strategies that the individual players will choose. We will focus mainly on noncooperative game theory as that is the basis for the Nash Equilibrium.

Understanding game equilibrium requires a discussion of solutions to games. Each solution is dependant upon which strategies are played, and there are many ways to go about choosing which particular pure strategy a player should play. One of these is to determine a player's *maximin value*. This means choosing the least "worst" outcome that could befall the player, in other words "cutting one's losses." Consider the following payoff matrix.

Figure 3: Example Payoff Matrix

		Player Two	
		C	D
Player One	A	4,0	-2,1
	B	2,0	1,2

Player one has the choice to play either strategy A or strategy B. Strategy A has the potential for the maximum payoff of 4 units; however, it also has the potential for the minimum payoff of -2 units. So, in order to remain safe and lose the least amount possible, player one will choose strategy B. Then, regardless of player two's strategy choice, player one will earn at least one unit. This guarantees player one a minimal

payoff level or the maximin value, also termed the *security level*. What is unique about this type of decision making is that it does not require the knowledge of the opponent's payoff. The player can make his own decision by simply picking his own best worst-case scenario.

Players may also decide which strategy to use depending on the opponent's decision. The player uses rational beliefs about the opponent's strategy to make his own. Obviously, this method does not prompt immediate action, as each player is dependent upon the opponent's decision. The game also requires that all of the payoff functions are common knowledge. Such a strategy combination that both optimizes the payoffs and is consistent with rational expectations concerning an opponent's strategy is termed *an equilibrium* of a game. Let's revisit the example payoff matrix.

Figure 4: Example Payoff Matrix

		Player Two	
		C	D
Player	A	4,0	-2,1
One	B	2,0	1,2

If player one plays strategy B first, then player two should play strategy D since his resulting outcome would be 2. If he had chosen strategy C, his outcome would have been 0. If player two chooses strategy D first, then player one should play strategy B resulting in 2. If she had chosen strategy A, her resulting outcome would have been 0. So the equilibrium of this particular game is (B, D) because neither player has an incentive to deviate from this outcome. There can be more than one equilibrium per game, but in this example, (B, D) is the only equilibrium.

In order to demonstrate that this particular game has only one equilibrium (B, D), let's test to see if outcome (A, C) is an equilibrium. Assume the game begins by player two choosing strategy C. In response, player one will choose strategy A, as that decision yields the maximum payout of 4 for her as opposed to only receiving 2 by choosing strategy B. At this point we see the "equilibrium" holds in one direction, but let's look at the other direction. If player one had chosen A originally, then player two would play strategy D, as that yields the maximum payout of 1 for him as opposed to only receiving 0 by choosing strategy C. So, because player two has an incentive to deviate from (A, C) by playing strategy D instead, (A, C) is not an equilibrium. We can test that the two remaining outcomes are not equilibriums in a similar manner.

The diagram below is a visual representation of the game with arrows.

Figure 5: Diagram of Equilibrium
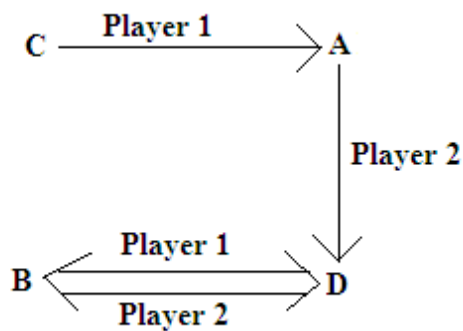
So, the only true equilibrium (B, D) can be seen in the above diagram by the double arrows between the two strategies. Regardless if player one had originally chosen strategy B or player two had originally chosen strategy D, the opponent's decision outcome would have been the same. That is how we know the strategy combination

(B, D) is an equilibrium, as the players get locked into those double arrows and have no

incentive to choose otherwise. Outcomes involving strategy C for example, are not an

equilibrium because the players have an incentive to deviate, which is represented by the

arrows moving them elsewhere. There are games in existence in which there is no

equilibrium when dealing with only pure strategies, and this is when mixed strategies

become crucial. There is a theorem by von Neumann which states "A game with finitely

many strategies has an equilibrium in mixed strategies,"[16] meaning that any game can

have an equilibrium if the pure strategies are modified into mixed strategies.

Another method to finding equilibriums is through *dominant strategies*. This

concept is best understood through an example. Consider the following payoff matrix.

Figure 6: Example Payoff Matrix

		Player Two	
		C	D
Player One	A	1,0	0,1
	B	2,0	1,2

If player one plays strategy A, then it would be in player two's best interest to play

strategy D. If player one plays strategy B, then it would still be in player two's best

interest to again play strategy D. By the same token, if player two plays strategy C, it is

in player one's best interest to play strategy B. If player two plays strategy D, again it is

in player one's best interest to play strategy B. With this in mind, player one should

choose to automatically play strategy B while player two should automatically choose to

play strategy D. This is because strategy D dominates strategy C, meaning every value in

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[16] Von Neumann, J. (1928) Zur Theories der Gesellschaftsspiele. *Mathematische Annalen* 100, 295-320.

strategy D is larger than its corresponding value in strategy C for player two. With the same logic, strategy B dominates strategy A. This is another way to determine which strategy to employ. It can be said with some confidence that a player will always choose to play a dominant strategy while no player willingly chooses to play a dominated strategy. This results from the initial game theory assumption that players make rational decisions and are always trying to maximize their potential outcomes.

One interesting flaw in using dominant strategies can be seen in an analysis of the Prisoner's Dilemma, one of the most notorious games used in game theory. Both the Prisoner's Dilemma game and the Battle of the Sexes game, which will be discussed in detail in a later chapter, are taken and modified from Luce and Raiffa's work *Games and Decisions* from 1957. In the Prisoner's Dilemma, two convicts are being held separately for a crime one of them committed. The guard, in an effort to make the criminals confess, strikes the same deal with both convicts. They can either choose to not confess (N) or confess (C), and the payoff matrix in Figure 6 shows the number of years either added to or subtracted from the sentence. Remember, positive outcomes are always represented by a positive number. For example, if prisoner one chooses to confess and player two chooses to not confess, player one will have three years subtracted from his sentence (a positive outcome, so it is represented by a positive number) while player two will have an additional year added to his sentence. Below is the complete payoff matrix for the prisoner's dilemma.

Figure 7: Payoff Matrix for Prisoner's Dilemma

		Prisoner Two	
		N	C
Prisoner One	N	1,1	-1,3
	C	3,-1	0,0

It is obvious that the dominant strategy combination is (C, C), meaning player one confesses and player two confesses. However, this equilibrium is in fact not optimal, as the strategy combination (N, N) would actually be more beneficial for both prisoners. This shows that in some cases rational behavior does not always lead to the optimal outcome. We now have a solid understanding of basic game theory and can begin an in depth look into the Nash Equilibrium specifically.

Chapter Two: The Nash Equilibrium

To begin our discussion of the Nash Equilibrium, a solution where no one has an incentive to deviate, let's first look at the following example: The Battle of the Sexes. In this game, a wife and husband are trying to decide what they will do for their evening entertainment. The wife prefers to go to the ballet, while the husband prefers to go to a football game. However, instead of going separate ways, both would prefer to spend the evening together than to spend it apart. Below is the payoff matrix for the game.

Figure 8: Payoff Matrix for Battle of the Sexes

		Husband	
		F	B
	F	1,2	0,0
Wife	B	.5, .5	2,1

First, notice that there is no dominant strategy for either the wife or the husband. We can see this by comparing all of a player's strategies to one another. For the wife, going to the football game will give her either a 1 or 0 payoff while going to the ballet will give her either a .5 or a 2 payoff. Since 1 is greater than .5 (the payoffs when the husband chooses to go to the football game), we begin by assuming that going to the football game is the dominant strategy for the wife. For this to hold true, the wife's payoff for going to the football game must also be greater than the payoff for going to the ballet when the husband chooses to go to the ballet. However, we can see than this is not the case, as 0 is not greater than 2. This means our assumption is false and going to the

football game is not a dominant strategy for the wife. In a similar manner, we can check that none of the other strategies are dominant for either the wife or the husband.

Also, notice that if both the wife and the husband reached their solutions by determining their *maximin* value, the wife would be going to the ballet earning at minimum .5 while the husband would go to the football game as his maximin value is also .5. This method of choosing a strategy by the players' maximin value results in the couple choosing a solution that is not optimal (.5, .5), when both (1, 2) and (2, 1) would be more beneficial outcomes for both the wife and husband. Instead, the couple should determine their solution by reacting to the other spouse's decision. This is, in essence, the Nash Equilibrium: "A strategy combination in which each player plays a best response to the opponent's behavior."[17] In this example, there are two Nash Equilibria, (F, F), meaning the husband chooses to go the football game and his wife follows him, or (B, B) meaning the wife chooses to go to the ballet and her husband follows her.

One problematic characteristic of the Nash Equilibrium is that at times it may cause dominated strategies to be played. Consider the following example.

Figure 9: Example Payoff Matrix

		Player 2		
		D	E	F
	A	1,1	0,1	0,1
Player 1	B	0,0	0,1	1,0
	C	1,0	0,0	1,0

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[17] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 84 Academic Press, New York. 1993.

It is clear that (A, D) is the only Nash Equilibrium of the game. However, when we use the dominant strategies method revisited in the previous wife/husband example we see that for player 1, strategy C dominates strategies A and B, while for player 2, strategy E dominates strategies D and F. This means the resulting solution, or final outcome, is (C, E), or (0, 0), which is obviously not the optimal solution of (A, D) or (1, 1).

One of the benefits of the Nash Equilibrium is that it is widely applicable to a large set of games. The Nash Equilibrium is a generalization of dominant strategy equilibrium. This means that any dominant strategy solution is a Nash Equilibrium, but not every Nash Equilibrium is a dominant strategy solution. This relationship makes the Nash Equilibrium widely applicable because many games do not have a dominant strategy, but they do have a Nash Equilibrium. However, some games exist that still do not have a Nash Equilibrium at all, such as rock, paper, scissors. If we return to the game's payoff matrix, we see that there is no equilibrium in pure strategies, as at every solution, one of the players has an incentive to deviate to another solution for an increased payoff.

Figure 10: Payoff Matrix for Rock, Paper, Scissors

		Player Two		
		Rock	Paper	Scissors
Player One	Rock	(0,0)	(-1,1)	(1,-1)
	Paper	(1,-1)	(0,0)	(-1,1)
	Scissors	(-1,1)	(1,-1)	(0,0)

A theorem exists that states the properties required for a game to have at least one Nash Equilibrium: 1) "the strategy set of each [player] is a nonempty compact and convex subset of Euclidean space" and 2) "the payoff function is a continuous function that is

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quasi-concave in a player's own strategy."[18] For a more mathematical discussion of this theorem, please reference Eichberger. Also, while I have demonstrated a concrete example of a convex set below, an example of a compact set is difficult to demonstrate without the necessary mathematical tools; please reference Stephen Abbott's *Understanding Analysis* pg. 84 for a clearer understanding of this concept.

As demonstrated earlier, the game rock, paper, scissors does not have a Nash Equilibrium. This is because the game has a finite number of pure strategies and "games with finite pure strategy sets lack convexity of strategy sets and quasi-concavity of payoff functions,"[19] which is necessary for Nash Equilibrium (see previous paragraph). To demonstrate how a set can lack convexity, consider the economic example of a consumer deciding which type of fruit to buy. The consumer has a son who only likes grapefruit and a daughter who only likes tangerines, and the consumer must choose only one fruit. However, the grocery store is only selling tangelos, a hybrid fruit which is 50% grapefruit 50% tangerine, on this particular day. Ideally, the consumer wants to mix her children's preferences. But she is now stuck because she is locked in to either buying grapefruit or tangerines, or in other words the set of fruit which includes only grapefruit and tangerines lacks convexity. If instead the set of fruit included grapefruit, tangerines, and every possible combination of the two fruits, then the set would be convex.

When the pure strategies are converted to mixed strategies, these games do satisfy the conditions necessary. Remember von Neumann's theorem "A game with finitely

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[18] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 90 Academic Press, New York. 1993.
[19] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 94 Academic Press, New York. 1993.

many strategies has an equilibrium in mixed strategies" presented in Chapter One. Now that we know that a game with mixed strategies satisfies all the necessary criteria for the existence of a Nash Equilibrium, the theorem becomes much more powerful and applicable. Now, we can generalize the original theorem dealing with the existence of Nash Equilibrium even further. A theorem by Irving Glicksberg states that any game that is a compact subset of a Euclidean space and has a continuous payoff function has at least one Nash Equilibrium in mixed strategies.[20] This means as long as mixed strategies are considered instead of pure strategies, the subsets no longer have to be convex, and the payoff functions do not have to be quasi-concave. So, in using mixed strategies instead of pure strategies, we do not have to fulfill as many of the requirements of the original theorem in order to reach a Nash Equilibrium. The less stringent requirements make a Nash Equilibrium must easier to find.

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[20] Glicksberg, I., (1952). "A further generalization of Kakutani's fixed point theorem". *Proceedings of the American Mathematical Society* 3, 170-174.

Chapter Three: Incomplete Information

One of the major assumptions of the Nash Equilibrium is that complete information is available. Remembering back from Chapter One, *games with complete information* exist when every aspect, including the number of players, the set of all nodes, the set of all actions, all information sets, and all payoffs, is considered "common knowledge," meaning that all of the players are fully informed of every aspect, and all the players are aware that the other players are also fully informed. This complete information includes the *type of players.* A player's type is composed of private information that only that specific player has access to, and this access begins at the very beginning of a game before any initial moves are planned. The complete payoff matrix is necessary for the players to make any sort of decision and hence critical to finding a Nash Equilibrium. Otherwise, the players would be blindly guessing or making assumptions as to which decision will produce the best outcome.

`   For example, reconsider a slightly revised version of the husband and wife example from Chapter 2. Now, the wife can choose to either go to the ballet, or go see a movie, and she values each activity equally. If she were to decide which activity the couple would do that evening, she would want to pick the solution that provided the greatest utility for both partners, meaning the combination that both she and her husband prefer most. However, if she does not know her husband's payoff for either activity, the wife is stuck and can only hope that a blind guess will yield the ideal solution. If she decides the couple will go to the ballet that night, yet the husband's value of going to the

ballet is actually (-3) while the value of going to the movie is 1, the solution is not ideal. The wife should have chosen the movie, but because she did not know the husband's payoff information, she was not able to make the best decision. In essence, "since players can no longer predict what would be a best response for the other players, they cannot determine what constitutes optimal behavior for themselves."[21]

The problem of incomplete information actually occurs quite frequently in the real world. Oftentimes it is up to managers to make difficult decisions with incomplete information and suffer the consequences. The missing payoff information is deduced from other known information and the probabilities of outcomes based upon historical data, but often the final decision comes down to a matter of instinct. One solution to the problem of incomplete information was proposed by J.C. Harsanyi in the late 1960's, in which games with incomplete information are transformed into games with imperfect information. The player facing incomplete information is seen as being uncertain of the type of player he will face as opposed to uncertainty about the payoff function. This assumes that certain types of players, who begin the game with differing private information, will behave in certain ways, producing different payoff functions. Next, "an artificial player, called *nature*, chooses according to some probability distribution the particular type of [opponent] that will play the game."[22] So the player can assume that the player he faces is a certain kind of player based on the probability distribution. The move of nature is unknown, so in this manner, the problem of incomplete information is

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[21] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 125 Academic Press, New York. 1993.
[22] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 125 Academic Press, New York. 1993.

transformed into imperfect information because the player does not know the previous move of nature. The Harsanyi solution of transforming games with incomplete information into games with imperfect information is based on some assumptions. First, all players are assumed to know all possible types of players they are facing, they just do not know which ones. Also, the probability distribution that is connected to each player type is also known. After nature chooses the type of each player, the individual player learns his personal type and what the probability distribution is and therefore has some insight about the types of the other players.

An example of this solution concept can be seen again with the husband and wife example. Initially the wife does not know which mood her husband is in, whether he feels like watching a movie or going to the ballet. In the same way, the husband is also ignorant of the wife's preferences. But the wife does know that her husband can be in either one of two moods. Assume there is a 60% probability that the husband is in the mood to watch a movie, meaning that his payoff to watch a movie would be higher than that to see the ballet. Then assume there is a 40% probability that he is in the mood to see the ballet which would result in exactly the opposite payoff. So, we now imagine that "nature" chooses which mood the husband is in based upon the specific probability distribution. The wife then decides to go to the movie based upon her own player type and the probability distribution of the player type of her husband.

This leads to the most well known solution concept, the Bayes-Nash Equilibrium, which is a refinement to the Nash Equilibria that overcomes the assumption of complete information. It is based upon Bayesian Decision Theory, which ties back to the earlier

discussion of players using utility functions or "decision functions" to decide which action produces the optimal outcome. In typical cases utilizing Bayesian Decision Theory, some element of information about the surrounding environment or the players is unknown, and this information is necessary for the players to choose the ideal action. This information is given in terms of signals. Many players begin a game uninformed of their own player type, so one important signal would inform a player of his own personal player type. The Bayes-Nash Equilibrium says that, in essence, each player forms contingency responses for each possible type of opponent and the respective Bayesian decision function.

Let's return to the husband and wife example. First, the wife learns her player type, the private information she has before the game begins including her preferences and beliefs about the environment. Then, she receives a signal from her husband; he came home from work raving about the new Band of Brothers Ballet and how badly he wanted to see it. The wife knows that typically her husband has a probability distribution of wanting to watch a movie 60% of the time and see the ballet 40% of the time, so traditionally she guesses that her husband wants to go to the movies. Yet, after receiving this signal, the wife has an inclination that the husband actually would prefer seeing the new ballet instead. These signal-contingent plans of action are a best response to the other player's decision functions. The key is that the Nash Equilibrium can be applied to these decision functions rather than a single strategy combination.

Chapter Four: Nash Equilibrium Refinements with Sequentiality

We know that there can be many different Nash Equilibria per game, and so a refinement centered on the sequence of moves was discovered which eliminates some of the less-than-ideal Nash Equilibriums. In order to analyze a game's sequence of moves, a process called *backward induction* is used. Backward induction works by looking at the last possible decision node (not terminal node) after which the chosen action will end the game. At this decision node, the player decides which action results in the best final outcome. Then, that decision node is transformed into a terminal node, and it is labeled with the payoff function from the previously chosen action. After repeating this process a finite number of times, the root node will be reached.

Consider the example revised from one given in Eichberger's *Game Theory for Economists*. This economic example analyzes the behavior between two firms. One is a large, well-situated firm that is a monopoly. This second firm is a small firm deciding whether or not to enter this monopolistic market. The second firm, called Firm Small, can decided to either enter (e) or not enter (ne) the market, and the first firm, called Firm Big, can choose to either accommodate (a) Firm Small or fight (f) Firm Small. Below is both the extensive and strategic form of the game.

Figure 11: Game Tree for Market Entry Example
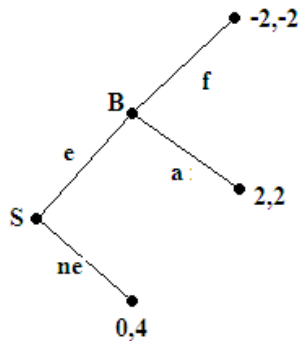
Figure 12: Payoff Matrix for Market Entry Example

		Firm Big	
		a	f
Firm Small	e	2,2	-2,-2
	ne	0,4	0,4

Notice that the Nash Equilibria are (e, a) and (ne, f) as neither player has any incentive to change from those outcomes. At first glance, it may appear that (ne, a) is also an equilibrium, but consider that outcome from Firm Small's perspective. Firm Small would have an incentive to switch from "not entry" which has a payoff of 0 to "entry" which has a payoff of 2, meaning that the solution (ne, a) is not a Nash Equilibrium.

Now, returning to equilibria (e, a) and (ne, f), both appear to be equally ideal outcomes in terms of best response strategies. However, after analyzing the sequentiality of moves, we will be able to show that one equilibrium is more realistic than the other. First, notice that the equilibrium (ne, f) can be viewed as a threat by Firm Big to keep Firm Small out of the market. However, if Firm Small deemed this threat to be non-

credible and choose to enter the market regardless, Firm Big would have to uphold the threat and fight Firm Small, resulting in a non-optimal solution. By analyzing the sequentiality of moves, we can predict that equilibrium (e, a) is indeed more favorable than (ne, f). We can now use backward induction to definitely show this. First, we look at the last decision node from which the resulting action ends the game: node B (decision node for Firm Big). Here, Firm Big can choose to either accommodate or fight Firm Small, and from the payoff matrix, we can see that accommodate Firm Small is the best decision for Firm Big. Then, according to the process of backward induction, we transform decision node B into a terminal node with payoff (2, 2). Let's analyze the new extensive form.

Figure 13: Extensive Form for Revised Market Entry Example
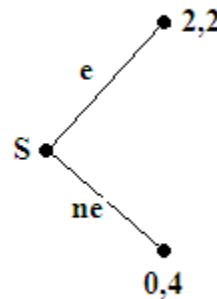
So now, the game consists of one decision to be made by Firm Small: either not enter with a payoff of (0, 4) or enter with a payoff of (2, 2). Judging from these two payoff functions, the optimal outcome for Firm Small is to enter the market, resulting in the final outcome of (e, a) which was the favorable Nash Equilibrium which was predicted by analyzing the sequentiality of moves.

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Backward Induction can only be applied to games with perfect information and finite extensive form. In 1965, R. Selten suggested a way to generalize the concept of backward induction to general games in extensive form using *subgame perfect equilibrium.* First, a *perfect equilibrium* means that the probabilities associated with every pure strategy are strictly positive, meaning greater than but not equal to zero. Also, the general rules of a Nash Equilibrium hold: that every strategy is a best response to an opponent's strategy.[23] A *subgame* is a part of an extensive form game tree that "must start at an information set with a single node and must contain all information sets that follow the initial node."[24] For an example of a subgame, consider the previous game tree in Figure 11, which has been copied below.

Figure 11: Game Tree for Market Entry Example

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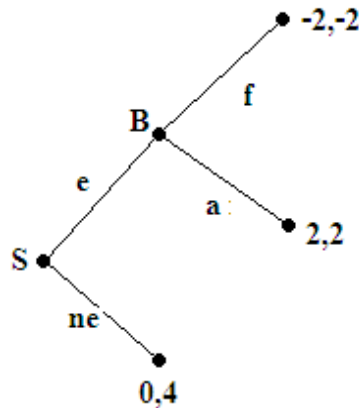
[23] Myerson, Roger. *Game Theory Analysis of Conflict.* Pg. 216. Harvard University Press, Mass. 1991.
[24] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 157 Academic Press, New York. 1993.

There are two subgames in this example. One subgame begins at the decision node B and includes both terminal nodes that follow. The second subgame begins at decision node S and contains all three terminal nodes that follow. Through this example, one can see that every game will have at least one subgame, since the one subgame can represent the entire game as a whole. Now, compare this to another example seen below.

Figure 14: Example of only 1 Subgame
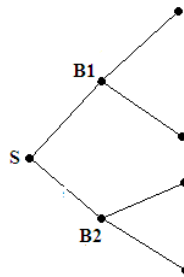
In this example, player S makes a decision, and then player B reacts. This example, which does not include any of the strategy nor payoff information for simplicity's sake, only has one subgame which is the entire game itself. We can see this when we try to find smaller subgames at either decision node B1 or B2. A subgame beginning at either of these nodes would not include the other B decision node, so the information set for player B would not be complete, which is necessary for a subgame. Because of this, the only subgame is the entire game.

When putting all the terms together, a *subgame perfect equilibrium* is a behavior strategy combination where the response of each player is a behavior strategy that is the best response to other player's behavior strategies. Using subgames lets us split a normal game apart, seeing if we can diminish the number of typical Nash equilibria by keeping

only those that are still perfect equilibria when applied to a smaller subsection of the game. Since some games have numerous Nash Equilibria, analyzing only the perfect equilibria can be a much more effective use of time and resources. Because subgame analysis does not require the use of terminal nodes, this procedure can be used on infinite games, unlike backward induction which requires a terminal node from which to begin the procedure.

While the Nash Equilibrium refinement of subgame perfect equilibrium does work for games with imperfect information, there is another refinement that is more effective for this particular type of game. This refinement is called a perfect Bayesian Equilibrium which is "a behavior strategy combination together with a belief system such that behavior strategies are equilibrium strategies at each information set given the beliefs, and beliefs are consistent with equilibrium strategies."[25] In order to understand this definition, we must first understand the concept of *beliefs.* Technically, a belief is a probability distribution set over the nodes of a particular information set. When we translate this technical definition into common terms, a belief is the degree of certainty a player has about the validity of information given views on the surrounding environment, historical data, and other relevant information. For instance, when analyzing a game in which a consumer who recently immigrated from Mexico decides to shop either at a Mexican market or a typical grocery store, we can use the Mexican immigrant's beliefs and preferences concerning Mexican food and culture to make an educated guess as to the immigrant's decision. A player's beliefs can be affected by many variables, including

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[25] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 167 Academic Press, New York. 1993.

environmental variables[26] which are details such as a player's height and weight that are typically omitted from a game as they are considered "non-game theoretic" details. However, these details may have an affect upon a person's beliefs.

In addition to beliefs, another concept which affects the probability of a specific equilibrium or strategy is the *focal point effect*, discovered by T.C. Schelling in 1960. This theorem claims that there are certain aspects in a player's life that may focus his attention on a particular equilibrium. Then, the player will begin to expect this equilibrium and because of this expectation, the player will ultimately fulfill it. In essence, the focal point effect creates a self-fulfilling prophecy. There are many aspects of a player's life that may lead to the focal point effect, and one of these aspects is culture. *Cultural norms* are "rules that a society uses to determine focal equilibria."[27] For example, when a consumer is choosing to purchase either a used Volvo Wagon or a Hearse, the typical consumer, due to the inherent connotation of death that our culture places upon a hearse, would choose the Volvo, even if the Hearse were cheaper, newer, and got better gas mileage.

Other aspects of life that may induce the focal point effect include *preplay communication*. An example of this can be seen in the husband and wife example from Chapter 3. The signal that the wife received from the husband regarding his preference to see the new Band of Brother's ballet over a movie makes the wife focus on the equilibrium of the couple attending the ballet together. Also, in examples of game

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[26] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 107. Harvard University Press, Mass. 1991.
[27] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 113. Harvard University Press, Mass. 1991.

communication which will be discussed more in depth in a later chapter, often there is a *focal arbitrator* who publically suggests a specific equilibrium to the players causing them to focus on it. In our world, this is known as marketing. For example, when a customer is deciding which brand of shampoo to buy, the typical American consumer is influenced by commercials and advertisements that she has seen on television, in magazines, or other forms of media. The advertisements and commercials are focal arbitrators that influence a player's decisions by causing them to focus on that specific product. Other aspects that might invoke the focal point effect include welfare properties such as equity and efficiency. Many people would choose a certain outcome over another if they feel it is a "fair" solution for the majority. Also, when dealing with repeated games, often an equilibrium that is composed of simplistic or stationary strategies is preferred. This can be easily seen in the real world, as people are creatures of habit. If two maximal outcomes exist, people will choose the outcome that is either the easiest to reach or is the most familiar.

All of these factors that influence the focal point effect including welfare properties, cultural norms, and preplay communication show that "game theory cannot provide a complete theory of human behavior without complementary theories from other disciplines."[28] This statement does not in any way suggest that game theory is any less meaningful in our analysis of human behavior; on the contrary, game theory is still an extremely important component of this analysis. It is just not the only approach that should be considered. We must remember this when dealing with not only game theory

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[28] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 114. Harvard University Press, Mass. 1991.

but any discipline that seeks to model human behavior. Connecting back to the selfish player discussion from Chapter One, this fact may explain why economists such as Elinor Ostrom assert that certain aspects of game theory do not explain the entirety of human behavior. If one is analyzing conflict with only a game theoretic mindset, then it does seem reasonable that players are inherently selfish and try to maximize their utility functions. However, when these scenarios are analyzed with other approaches as well as game theory in mind, cultural norms and other beliefs trigger the focal point effect. This causes the players to perhaps value community advancement over personal gain.

Returning to the previous discussion of beliefs in connection to the Perfect Bayesian Equilibrium refinement, if a player's beliefs allow him to assign a probability to each node then, "he could determine the expected payoff of any behavior strategy combination, conditional on having reached this information set."[29] There exists one major assumption concerning the *system of beliefs* that is applied to each information set; the system of beliefs is common knowledge, meaning all players have the same beliefs and every player knows the other players hold the same beliefs.

We now have an understanding of three different refinements of the Nash Equilibrium: backward induction, subgame perfect equilibrium, and Perfect Bayesian Equilibrium. We will now analyze a well-known example from economics that illustrates the Perfect Bayesian Equilibrium. The following modified example is based on one from Eichberger's *Game Theory for Economists.* The original "market for lemons" concept was introduced in 1970 by G. Alkerloff in his article in the *Quarterly*

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[29] Eichberger, Jeurgen. *Game Theory for Economists.* pg. 165 Academic Press, New York. 1993.

Journal of Economics. The market for lemons example can be easily illustrated by the consumer market for used cars. Typically, when a consumer shops for a used car, he does not know for certain whether or not the car is of high or low quality. So, there is a certain probability that the specific car in question is of either high or low quality. Now, the seller always knows the quality of the vehicle, while the consumer does not. This is an example of asymmetric information.

There are two types of scenarios that can occur. In the first scenario, the consumer is faced with two different sellers who are offering cars for different prices. In this scenario, the consumer, given his belief system, will choose the more expensive car, as he assumes that the more expensive car is the one of higher quality while the less expensive car is the one of lower quality. However, when analyzing the game before all decisions are made, the seller of the lower quality car knows that if the prices are differentiated, the consumer will chose the more expensive car, thus giving the poor quality seller an incentive to match his price to that of the high quality seller. This leads us to scenario two in which both cars are offered at the same price. If the price was that of the low quality car, the consumer would recognize this and not accept the offer as he assumes that he would be receiving a low quality car. The consumer is making this assumption because a seller with a high quality car would not be willing to sell at such a low price, as the seller knows he could sell at a higher price in another market. If the price was that of the high quality car, the consumer will accept the offer assuming that this should be the high quality car. However, since both high quality and low quality cars are sold at the same price, the only way the buyer will act on this assumption is if the

majority of used cars are of high quality. This demonstrates that there is a perfect Bayesian Equilibrium of both sellers pricing their cars at the high quality price if the average quality of cars is at a high enough level. If it is not, then no cars will be traded.

This version of the market for used cars varies slightly from that in real life where often the consumer assumes the quality of the car is simply average and therefore is only willing to pay the price for an average car as opposed to the price for a very high quality used car. This makes selling a very high quality used car in this market illogical, as the price offered to the seller will not be high enough to make selling the car worthwhile. This drives the high quality cars out of the used car market, and the average quality of used cars for sale drops as a result. As one can see, the asymmetric information between the uninformed buyer and the all-knowing seller causes the market to collapse, as eventually all of the cars with even a hint of quality will be driven from the market. The problem of asymmetric information is a common thread that helps explain market collapses.

Chapter 5: Games with Communication

There are many realistic situations where players can communicate to one another either during or before a game. For example, a buyer and a seller of a house will negotiate through their realtors to decide on an agreeable price that both parties can accept. One way to go about portraying the possible outcomes that are affected by communication is to build the communication directly into the game by representing each interaction as a different node with many possible actions. However, it is quickly obvious that trying to represent a game with communication in this manner becomes extremely complicated, as every different word or phrase spoken between players can evoke a different feeling or portray a different meaning. The games would be unmanageable at this level. Instead, we break games with communication down by type and analyze the resulting solution concept with the results of the different types of communication built into them.

First, let's look at *games with contracts*. Signing a *contract* forces the players to implement a specific correlated strategy to receive a specific allocation or payoff. A *correlated strategy* "is any probability distribution over the set of possible combinations of pure strategies that these players can choose."[30] In many games, the players are not required to sign the contract, and so possible game strategies must include not only accepting but also declining the contract. Consider the following scenario.

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[30] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 247. Harvard University Press, Mass. 1991.

Figure 15: Payoff Matrix Example with Contracts

		Player 2		
		x2	y2	s2
	x1	(1,1)	(0,6)	(0,6)
Player 1	y1	(6,0)	(2,2)	(1,1)
	s1	(6,0)	(1,1)	(3,3)

In this game, player 1 and player 2 are faced with three different strategy options. They can choose to play either strategy x or strategy y or sign a contract. For both players, strategy x is a dominated strategy, meaning the players will choose between strategy y and signing the contract, neither of which are dominated. However, for both players there is an incentive to sign the contract. While the highest payoff possible for either player choosing either y or s is 6 and the lowest payoff possible is 1, signing the contract has a higher overall outcome of 3.33 which is equal to (6+1+3)/3, while strategy y only has an overall outcome of 3 which is equal to (6+1+2)/3. So, both players have an incentive to sign the contract, and indeed, it would be most beneficial for one player to sign the contract if he thought the other player was going to. On the other hand, if player 1 thought player 2 was not going to sign the contract and instead choose strategy y, it would be more beneficial for player 1 to not sign the contract and instead choose strategy y also. This example shows that players can decide to sign or not sign a contract based upon the other player's decisions. So, we have a Nash Equilibrium found in games with contracts.

In some games, a *mediator* exists to facilitate the communication between players. The mediator collects all of the information about the game and then offers recommendations to each player as to which strategy she should implement. It is

assumed that communication between the mediator and player is one-way, meaning the mediator gives his recommendation to the player and then communication stops; the player is not allowed to question the mediator as to other recommendations. This is often the case in real life when lawyers or an acquisition company are used to communicate between firms and often times recommend courses of action. This next example shows the usefulness of mediators and how, many times, it is in the players' best interest to follow the mediator's recommendations. Consider the payoff matrix below.

Figure 16: Payoff Matrix Example with Mediators

		Player 2	
		x2	y2
Player 1	x1	(3,1)	(0,0)
	y1	(2,2)	(1,3)

In this scenario, there are three equilibria (x1, x2), (y1, y2) and a third based on a randomized equilibrium of (3/2, 3/2). The randomized equilibrium exists when players do not know the decision of the other player and therefore base their playoffs as to the expected outcome of the game, meaning (3+0)/2 if they play strategy x or (2+1)/2 if they play strategy y. Now consider what happens when we introduce a mediator to this scenario. Let's say the mediator recommends player 1 to play strategy x1. Now the mediator's role is to offer the best recommendation to both parties, so if the mediator recommends player 1 to play strategy x1, then player 1 knows that player 2 was recommended to play strategy x2 since by playing strategy x2, player 2 receives a payoff of 1, as opposed to receiving a payoff of 0 if he had played strategy y2. So player 1 has every motivation to follow the mediator's recommendation. Now, let's say that the

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mediator recommended that player 1 play strategy y1. This means that the mediator recommended that player 2 play either strategy x2 or y2 with equal probability. Again, because the mediator is recommending the best outcome for *both* players, (2, 2) and (1, 3) both have an overall expected outcome of 2 since (2+2)/2 and (3+1)/2 both equal 2. This means the two outcomes seem equally ideal to the mediator, and so there is an equal probability that the mediator will recommend either of these outcomes to the players. So, since player 1 does not know which outcome player 2 was recommended to play, player 1's expected outcome is 3/2 which is equal to (2+1)/2 which is the same expected outcome if he switched to strategy x1 or 3/2 which is equal to (3+0)/2. So, again player 1 has every incentive to follow the mediator's recommendations and not switch. We use a similar argument to say that player 2 has no incentive to deviate from the mediator's recommendations.

Through this analysis, we know that the mediator will never recommend strategy (x1, y2) as there are better outcomes for both players. So, we can represent the possible recommendations of the mediator in another payoff that removes strategy (x1, y2). Now, the remaining three equilibria (x1, x2), (x2, y1), and (y1, y2) are chosen at random by the mediator with an expected outcome of (2, 2) resulting from (3+2+1)/3. This randomized payoff is obviously higher than the original randomized payoff of (3/2, 3/2) that still included the strategy (x1, y2). This example shows the usefulness of a mediator in a game with communication as he was able to increase the overall expected outcome of each player from 3/2 to 2 by removing the less than ideal payoff of (x1, y2) from his possible recommendations. This example also demonstrates the use of a *correlated*

equilibrium which is any correlated strategy which is "self-enforcingly implemented with the help of a mediator who can make nonbinding confidential recommendations to each player."[31] The *self-enforcing* nature of this equilibrium means there is nothing that binds the players to this recommendation such as a contract other than the player's own personal motivation to maximize his outcome.

Bayesian games with communication require additional analysis. Remember, in Bayesian games, some information is private, such as a player's type. This would create two-way communication between the mediator and the players, as the players would tell the mediator their types and the mediator would respond with the appropriate recommendation taking this information into account. Because of this two-way communication, there is an opportunity for the player to lie about his or her type, as well as to not follow the mediator's recommendation. We say that a Bayesian game with communication has a mediation plan that is *incentive compatible* if and only if "it is a Bayesian Equilibrium for all players to report their types honestly and to obey the mediator's recommendation when he uses the mediation plan."[32] However, when a mediation plan is not incentive compatible, rational and intelligent players are not likely to act honestly or obediently. We can generalize this notion of incentive compatible mediation plans with the *revelation principle* for general Bayesian games. This principle states that "given any general communication system and any Bayesian Equilibrium of the induced communication game, there exists an equivalent incentive-compatible

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[31] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 253. Harvard University Press, Mass. 1991.
[32] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 260. Harvard University Press, Mass. 1991.

mediation plan, in which every type of every player gets the same expected utility as he would get in the given Bayesian Equilibrium of the induced communication game."[33] This means that we can use a mediator in a game with players who respond honestly and obediently without a loss of generality between Bayesian games.

The notion of incentive compatible mediation plans encourages a discussion of two topics that are inherently built into it. To avoid *adverse selection*, we "need to give players an incentive to report information honestly." To avoid *moral hazard,* "players [need] an incentive to implement recommended actions."[34] These two needs can be easily understood with a health-care insurance company example. Health-care insurance companies want to know which customers have which medical conditions, because these customers will be more expensive to insure. However, customers have an incentive to lie about their existing health problems, because then the insurance company will charge them a lower premium due to the customer's supposed good health. The insurance company needs to encourage honesty, so that it can appropriately estimate its potential losses. Moral hazard can also be illustrated using health insurance. For example, if a customer is insured for every type of medical care possible, then he will be less cautious in his everyday life because he knows his medical expenses will be completely paid for. Customers with every medical expense covered by insurance will be far less cautious than customers with less medical coverage. The health insurance company also has an

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[33] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 260. Harvard University Press, Mass. 1991.
[34] Myerson, Roger. *Game Theory Analysis of Conflict*. Pg. 263. Harvard University Press, Mass. 1991.

incentive to prevent moral hazard by encouraging people to follow its recommendations and be as cautious as if they had less than adequate medical insurance.

Moral hazard in the health insurance industry also causes customers to over utilize procedures and be less cost conscious. For example, a customer who gets an MRI does not care if the procedure costs $5,000 or $500. The insurance company will pay regardless. This overutilization and lack of customer cost awareness creates an extremely expensive and inefficient situation for insurance companies.[35]

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[35] Nyman, John. "Is 'Moral Hazard' Inefficient? The Policy Implications Of A New Theory." *Health Affairs,* 23, no. 5 (2004): 194-199.

Chapter 6: Housing market collapse analysis

We will now analyze the housing market collapse of 2007 using the notion of adverse selection and moral hazard in connection with Bayesian games with communication. With our understanding of basic game theory, the Nash Equilibrium, its refinements in Bayesian games, and now Bayesian games with communication, we have the building blocks to understand exactly what went wrong in this market and to suggest several possible solutions. Before we delve into this discussion, here is a brief overview of the housing market collapse.

One of the largest problems in the mortgage market was brokers writing loans for risky customers. It began in the 1990's when the requirements for mortgages were relaxed and the "subprime" mortgage market boomed. A subprime mortgage is a loan to someone with a poor credit history; it is a risky, but possibly very rewarding, investment due to the high interest rate charged the borrower. Lower mortgage standards in the subprime market caused the "homeownership rate [to increase] from the 64 percent range of the 35 years before 1995 to an all-time high of 69 percent in 2004."[36] In many cases, the mortgage brokers did not care about the risk because knew they were going to bundle up many of the loans into financial products called "mortgage backed securities" and sell them to other financial entities. If the mortgage brokers kept the loans, they assumed that if the loans ever did go bad, the collateral would be worth more than the loan balance

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[36] Ronald D. Utt, Ph.D., Executive Summary: The Subprime Mortgage Market Collapse: A Primer on the Causes and Possible Solutions 4/22/08 *Thomas A. Roe Institute for Economic Policy Studies at The Heritage Foundation. http://www.heritage.org/research/economy/bg2127es.*cfm

meaning the borrower could sell the house to pay off the loan. Finally, many of the customers were also to blame as they were dishonest about their credit history and financial stability. There was a lack of due diligence by mortgage brokers and the rating agencies who rated the mortgage-backed securities much higher than they should have. If any of these people had taken a closer look at the loans that were being made or reconsidered the unrealistic assumption that house prices would continue to rise, many of the riskier loans would not have been written, as the customers would have been considered far too risky. Also, the rating agencies would not have rated the mortgage-backed securities as high as they did, making the market for these securities much more realistic. However, either no one seemed to have enough at stake to practice due diligence or all parties seriously misjudged the future of the housing market.

Even if the customers were risky, there would only be a problem if housing prices fell, and everyone believed they would not. This fatal assumption is what brought down the metaphorical "house of cards." Housing prices began to fall in 2006. Soon, homeowners found themselves owing more for their homes than they were actually worth. This caused many homeowners to simply walk away from the debt, reducing net worth on many investors' balance sheets and driving prices of these mortgage-backed securities down. Tonko Gast, the Europe CEO of Dynamic Credit Management a company which not only manages clients' credit but also offers financial advice and analysis, "estimates that most of AAA rated mortgage-backed CDO's [collateral debt obligations] that the industry created since 2006, are now worth less than half their value.

Some are worth close to zero."[37] Collateral debt obligations are simply pools of

mortgage backed securities. Soon, the ripples from the falling home prices, the declining

value of the mortgage-backed securities, and the bad assets on lenders' books spread to

financial markets across the world. Eventually, a credit crunch put the U.S. in a recession

in December of 2007.

So, what really went wrong? If we take a closer look at how a mortgage broker

evaluates a customer, this is a Bayesian game with communication which we learned

about in Chapter 5. Both players have a specific type that is, for the most part, unknown

to the other until information is shared. The customer may be a financially stable

individual with a high credit score, or he could be financially unstable with a low credit

score, making him a risky borrower. The mortgage broker may also be extremely liberal

in his loan writing or he may be very conservative and only write loans for very safe

customers. However, as the housing bubble inflated, mortgage brokers did not have an

incentive to care about the types of the customers because the brokers were simply

writing the loans and then selling them off, completely removing the risk from their

books. One of the problems in this scenario is the asymmetric information between the

customer and the broker. Before lending standards were relaxed, it was very important

for the broker to know exactly what type of player the customer was. Until the customer

revealed this, the broker was left guessing and was forced to make decisions based on

assumptions, much like the market for lemons scenario described in Chapter 4. After

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[37] "This American Life Episode Transcript." Program 355. *National Public Radio.*
http://www.thislife.org/extras/radio/355_transcript.pdf

lending standards were relaxed, brokers could easily sell loans removing the asymmetric information problem. It was no longer important to the broker what type of player the borrower was.

Since asymmetric information was not the major issue in causing the housing market collapse, let's analyze adverse selection. The risky customer has an enormous incentive to be dishonest about his player type so that he receives a larger loan. Not only did the mortgage brokers do nothing to encourage honest behavior from the customers, they almost encouraged the dishonesty by loosening their standards and not doing the due diligence necessary. The adverse selection that was present in this Bayesian game with communication was not addressed, and so risky mortgages were written to customers who could not afford them. For example, in Sonoma County, California, "when home prices peaked in 2005, the typical home buyer in Sonoma County claimed to earn $120,000 a year on loan documents, according to federal home loan data. But they actually earned about $80,700, according to Census data. The spread grew in 2006, when the typical buyer claimed to earn $132,000; their actual income was about $79,000."[38] This is just one example of adverse selection found during the subprime mortgage boom.

In addition to adverse selection, the housing market crash also illustrates moral hazard. The mortgage brokers did not act with as much caution as they should have because they knew the risk was going to be transferred to another financial entity or the government, much like the health-insurance customers who know that all of their medical

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[38] Coit, Michael. "Loans Built on Lies." *The Press Democrat* (Sonoma County, Calif.), 2/10/08 http://*www1.pressdemocrat.com/apps/pbcs.dll/article?AID= /20080210/NEWS/802100349/1033/NEWS01*

bills will be paid by the insurance company. The brokers did not suffer any

consequences for writing risky loans. Mortgage brokers had no incentive to follow basic

financial logic, and so they ignored risk.

With either adverse selection or moral hazard, players on both sides acted with

"dishonesty" and "disobedience" to the rules of economic markets. This means that the

mediation plan, if one existed, was not *incentive compatible*, as players benefited by

acting dishonestly. To eliminate this there would have to have been a meditation plan

that was incentive compatible. Below is an example payoff matrix that is a basic

representation of the expected outcomes for the borrowers and the brokers.

Figure 17: Payoff Matrix for Mortgage Lending

		Broker	
		Lend	Deny
Borrower	Dishonest	6,4	(-1,1)
	Honest	3,3	0,1

First, let's analyze each payoff outcome. If the borrower is dishonest **(D)** and

inflates his income level, he will be able to receive a larger loan, thereby allowing him to

purchase a larger and nicer home. This explains why the borrower has an expected

payoff of 6 if he is dishonest while the expected payoff is only 3 if he is honest **(H).** If he

is denied **(Dn)** the loan, his payoff is zero if he is honest. If he is dishonest and does not

receive the loan, the broker has discovered the dishonesty, causing further penalty to the

borrower, hence a (-1) payoff. John Falk of the National Association of Mortgage

Brokers reports that "lying on a mortgage application is a federal crime . . . [and] can

result in jail time."[39] This possible jail time results in the negative one payoff for the borrower if he is caught. The broker, on the other hand, will receive the payoff of 4 if he lends **(L)** to a dishonest person because the loan amount is greater, and he does not have to deal with the consequences if the borrower defaults. If he lends to an honest person, the broker will receive only 3. If he chooses not to lend, he will only receive the fees for making his evaluation.

First, let's analyze this payoff matrix and find an ideal solution using the maximin method we learned in Chapter 1. If the borrower is trying to minimize his losses, he will choose strategy **H** as the least he can lose is 0 while if he chooses strategy **D** he could lose 1. The broker will choose strategy **L**, as he loses nothing from either strategy and lending offers the highest payoff. In previous discussions of the maximin payoff method, we declared this method was less than ideal, as often times the solution was not optimal. This happens here. The resulting solution is **(H, L)** which has a payoff of (3, 3). For the players, the optimal Nash Equilibrium is instead **(D, L)** which has a payoff of (6, 4).

However, while the solution **(D, L)** may be optimal Nash Equilibrium for the players, the dishonesty by the borrower as well as the lack of due diligence by the broker is the behavior that fueled the housing market crash. This is the outcome we as a society want the borrowers and the lenders to avoid. Having the borrowers utilize the maximin method when choosing a strategy seems ideal as it results in the type of behavior that is best for society as a whole. However, typically players who utilize the maximin method are risk averse, trying to minimize their losses instead of maximizing their gains. There

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[39] Know, Noelle. "10 Mistakes that Made Flipping a Flop." *USA Today* 10/22/06

certainly were many borrowers who followed this method during the housing market crash and were not trying to cheat the system. Yet, the borrowers who chose to be dishonest were certainly not risk averse and therefore did not choose the maximin method. Also, as discussed in Chapter 4, a player's decision is often times influenced by a focal point, typically beliefs based upon cultural norms. In this case, the dishonest players could be influenced by the stereotypical American cultural norm that "bigger is better," and this belief would then trigger the focal point effect, making them more inclined to choose dishonesty over honesty.

Instead of evaluating the payoff matrix using the maximin method, let's look for dominant strategies, also discussed in Chapter One. Looking at the strategies, strategy **H** for the borrower is not dominated by strategy **D**, so the borrower will base his decision on the broker's. For the broker, however, strategy **Dn** is dominated by strategy **L**, so in every scenario, the broker will lend to the borrower. This is due to the lax lending standards of the time as well as the shifting of consequences through the mortgage-backed securities to other financial entities. So, the only Nash Equilibrium is (**D, L**) where the borrower is dishonest and the broker lends, thereby creating the problems of adverse selection and moral hazard as previously discussed.

How do we assure that neither player benefits by being dishonest? First, the original lending requirements should not have been lowered. Risky customers should not have qualified for mortgages in the first place. However, this would not have stopped some from lying outright about their player type. One way to discourage dishonest behavior is to implement a disincentive to lie. But, when we analyze this using game

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theory, we see this does not solve the problem. Let's look at another example payoff matrix.

Figure 18: Payoff Matrix for Mortgage Lending with Borrower Disincentive

		Broker	
		Lend	Deny
Borrower	Dishonest	6,4	(-7,1)
	Honest	3,3	0,1

This payoff matrix is identical to the previous one in Figure 15 except there is an additional penalty to the borrower if he is caught being dishonest. However, the same strategies are still dominated as in the previous payoff matrix, and the additional disincentive does not change this. Strategy **Dn** is still dominated by strategy **L** for the broker. So, even with the disincentive for the borrower, the broker will still lend, regardless of the player type. This means the only Nash Equilibrium is again (**D, L**), and the problem of moral hazard and adverse selection remains unsolved.

Even today in 2010, there is still dishonest behavior. For example, in the mortgage modification process customers are trying to renegotiate their mortgages to receive lower interest rates or an extension on payments. If customers understate their earnings by less than 25%, and thereby receive larger mortgage modifications, they are not required to restart the mortgage process, but simply edit their applications and continue with the process.[40] By not forcing the dishonest applicants to completely restart the mortgage modification process, there is no incentive for the customers to be honest.

[40] Beck, Rachel. "No consequences for lying mortgage borrowers." *Seattle Times*. 12/28/09

There must be incentives to persuade people to be honest; otherwise the behavior

perpetuates itself and adverse selection continues. Credit scores are a way to achieve this

honesty, but they only work if the brokers and rating agencies actually use them to

appropriately evaluate the loan applications.

Imposing a disincentive upon the borrower does not solve the problem of moral

hazard or adverse selection. However, a disincentive for the broker who makes bad loans

makes the strategy to deny to loner dominated by the strategy to lend. One disincentive

is to force the brokers to keep a percentage of the loans that they write. This forces the

brokers to "eat their own cooking." If they write risky loans that default, they will lose

money. This idea finally imposes a consequence upon the broker for not using due

diligence, as the brokers can no longer sell off all of the loans they create and escape the

consequences. Let's look at a final payoff matrix to analyze this new scenario.

Figure 19: Payoff Matrix for Mortgage Lending with Broker Disincentive

		Broker		
		Lend Success	Lend Default	Deny
Borrower	Dishonest	6,4	(-5,-3)	0,1
	Honest	3,3	(-1,-2)	0,1

There are several differences between this payoff matrix and the previous ones. First, the

broker can now choose between three difference strategies. He can choose to lend with

the outcome resulting in success meaning the borrower makes his payments successfully

(LS). The broker can also choose to lend with the outcome resulting in a default for the

borrower **(LD).** Finally, the broker can deny the loan all together **(Dn).** The borrower is

faced with the same two options: to be dishonest **(D)** or honest **(H).** Now, when we look

at the actual payoffs, we see that again the strategy **LS** for the broker dominates his other

two choices, so at first glance it appears that we have not solved our problem. However,

does the broker really have a choice as to whether his loan results in a success or a

default? While the broker can increase his probability of success through due diligence,

ultimately this is not a choice that he can make. So this payoff matrix functions a bit

differently than the ones we have seen in the past. Instead, there is a distribution on the

two outcomes (**LS and LD**) that depends on the player type. For instance, if the player is

dishonest and lies about his income, there is a far greater probability that he will default

on his loan than someone who is honest. For instance, from the broker's perspective, an

honest borrower may default on his loan 5%[41] of the time, while a dishonest borrower

may default on his loan 10% of the time. These percentages also take into account

economic factors that the previous payoff matrices did not account for, implying an

increased market awareness of the brokers. One of the fatal assumptions in the housing

market crash was that housing prices would continue to rise, an assumption that

challenges the basic principles of supply and demand. Now, this probability distribution

encompasses not only inherent risk as to the type of borrower, but also inherent risk in

the market including house prices declining and/or interest rates rising. Increasing

interest rates affect adjustable rate mortgages, a significant number of subprime

mortgages were ARM's.[42] Both of these economic factors increase default rates. So, in

reality the broker simply decides to lend or deny, and the probability distribution placed

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[41] This percentage is unrealistic. Today, the foreclosure rate is around 2.8% while before that the foreclosure rate was below 1.5%. The larger numbers are used for convenience and simplicity.
[42] "What is a Subprime Mortgage?" *Investopedia* a Forbes Digital Company. www.investopedia.com

upon **LS and LD** decides the rest. Because of this probability distribution, **LS** is no longer the dominant strategy. Instead, if the possibility of default is high enough and the penalty to the broker is high enough, this encourages more due diligence by the broker. There is no longer only one Nash Equilibrium, but instead outcomes based upon an inherent probability distribution.

Not only should the brokers have to keep a portion of their loans so that they experience consequences of making bad loans, but the ability of mortgage brokers to gather up risky loans and sell them to others needs to be regulated to a larger degree. Brokers should not get away with creating such risky loans and selling them to others. This requires that the rating agencies rate the loans and mortgage backed securities accurately so that the financial entities buying these loans know what they are buying. Also, the buyers of mortgage-backed securities should not rely on the government to bail them out when the loans backing these securities fail. If the buyers knew they would be held responsible for their purchases, they would have been more cautious. Finally, the entire real-estate market should heed the lessons learned from the past: housing prices do not always rise. This fatal assumption gave everyone who touched the risky loans the confidence that they would not fail, and therefore caused suboptimal behavior. Everyone knows that markets tend to be cyclical. Assuming that a market will never fall is a blatant disregard of historical lessons and economic teachings.

The housing market crash resulted from both dishonesty and disregard for classic historical lessons. Adverse selection and moral hazard created an environment that encouraged dishonesty, and the lack of lending standards as well as a lack of due

diligence by both brokers and rating agencies created a market bound to fail. As soon as the fatal assumption of ever-increasing housing prices proved false, the entire market collapsed. By using the concepts of adverse selection and moral hazard found in Bayesian games with communication as well as asymmetrical information found in perfect Bayesian Equilibrium, we are able to understand what went wrong with the housing markets and what could help solve the problem. By analyzing the dominant strategies found in this particular game, we showed that a disincentive for the borrower would not be effective, as the broker still has a dominant strategy to always lend, regardless of the borrower type. We also showed that a disincentive for the broker would increase the likelihood of honest behavior as long as the broker understood the probability distribution placed upon the success or default of a loan. Through increased due diligence, the broker could increase the probability that the loan would be successful, but this only matters if the disincentive for writing a poor loan is large enough.

Understanding the basic definitions, forms, and methods to finding a Nash Equilibrium, along with the varying refinements and types of games including those with incomplete and imperfect information, and games with communication proved useful in analyzing the housing market collapse. However, despite all of the building blocks learned in the previous chapters, analyzing a scenario from a game theoretic approach tells one part of the story. A complete analysis includes many different perspectives, each with its own set of assumptions and guidelines. In game theory, players are rational and intelligent. Game theory recognizes that many outside forces contribute to preferences reflected in utility functions. But there are many other disciplines that focus

on these outside forces. A more complete picture incorporates these other disciplines. Game theory is just one tool to understand the world around us and how humans interact within it.

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