Fall 2007

Trigonometry Unit Based On Brain Research

Cynthia Tait
Regis University

TRIGONOMETRY UNIT

BASED ON BRAIN RESEARCH

by

Cynthia Tait

A Research Project Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Education

REGIS UNIVERSITY

July, 2007

ABSTRACT

Trigonometry Unit Based on Brain Research

The purpose of this project was to provide the teachers of high school geometry students with a unit on trigonometry that uses strategies that work with the natural brain process to promote successful learning. The trigonometry unit also serves as a sample unit in which brain compatible teaching strategies have been applied. Teachers can increase their effectiveness by transferring the strategies discussed in this project to other mathematics units, other subjects, or other grade levels. The unit was developed based on the findings from the current research on: (a) brain physiology, (b) adolescent learning, and (c) self-efficacy. A review of literature shows that researchers have identified processes that promote and impede the ability of the brain to learn successfully. Lesson plans and graphic organizers have been provided. The project was reviewed by education professionals and their comments along with the limitations of the project and suggestions for further research are discussed in chapter 5.

TABLE OF CONTENTS

Chapter Page

1. INTRODUCTION 1
   Statement of the Problem 1
   Purpose of the Project 2
   Chapter Summary 2

2. REVIEW OF LITERATURE 4
   Brain Research 5
      History of Brain Research 5
      The Brain 6
      Tools Used for Brain Research 9
      Brain-Based or Brain Compatible Learning 9
      Windows of Opportunity and Brain Plasticity 14
      Natural Learning Process 15
      Neurodevelopmental Systems 19
   The Adolescent Brain 20
   Self-Efficacy 22
      Methods to Strengthen Self-Efficacy 24
   School Environment 26
   Adolescent Goals 27
      Delinquent, At-Risk, and Not At-Risk Youth 28
      Perfectionists 29
      Efficacy and Goals 29
   Motivation 31
   National Council of Teachers of Mathematics Standards 33
   Colorado Model Content Standards 34
   Chapter Summary 35

3. METHOD 37
   Targeted Audience 37
   Goals of the Applied Project Procedures 38
   Procedures 38
   Peer Assessment 38
   Chapter Summary 39

4. RESULTS 40
   Lesson Plan #1 NHLP 41
   Classroom Environment to Promote Brain Compatible Learning 45

The Trigonometry Unit 46
Unit Plan for Trigonometry 48
Lesson Plan #1: Similar Right Triangles 50
Lesson Plan #2: Pythagorean Theorem 53
Lesson Plan #3: Special Right Triangles 56
Lesson Plan #4: Trigonometric Ratios 59
Lesson Plan #5: Solving Right Triangles 62
Lesson Plan #6: Clinometer Activity 65
Lesson Plan #7: Unit Assessment 68
Additional Extensions 70
Chapter Summary 70

5. DISCUSSION 71
Introduction 71
Goals Successfully Incorporated into the Project 71
Assessment and Strengths of the Research Project 72
Limitations of the Research Project 74
Recommendations for Additional Research 75
Research Project Summary 75

REFERENCES 76

APPENDICES
A. Three Types of Equipment Used in Brain Research 79
B. Neurodevelopmental Systems as They Translate into the Aims of Education 81
C. What One Would See within Schools for All Kinds of Minds 83
D. Colorado Model Content Standards for Mathematics, Grades 9-12 85
E. Learning Scene Investigators and Major points About Learning Handouts for NHLP Lesson 88
F. Similar Right Triangles 91
G. The Pythagorean Theorem 101
H. Special Right Triangles 107
I. Trigonometric Ratios 113
J. Solving Right Triangles 122
K. Clinometer Activity 129

LIST OF FIGURES

Page

1. Direction of Impulse through a Neuron 7

2. Diagram of a Neuron: The Myelin Sheath 7

3. Diagram of the Brain: Terms Used to Describe Parts of the Brain 8

Chapter 1

INTRODUCTION

Relatively recently, the findings from research conducted by neuroscientists, (Calvin & Ojemann, 1994; Edelman, 1992; Goldblum, 2000; Greenfield, 1997; Khalsa & Stauth, 1997; Morowitz & Singer, 1995; Ratey, 2001; Restak, 1994; Rose, 1992; Thompson, 1995; all cited in Smilkstein, 2003) have supplied educators with a wealth of information on the physiology of the brain. These researchers have determined that one of the main functions of the brain is to learn, and they have identified processes that promote and impede the ability of the brain to learn successfully. Educational researchers, consultants, and professors (Caine & Caine, 1997; Gunn, 2007; Jensen, 1998; Smilkstein; Sylwester, 1995; Wolfe, 1998) have interpreted these neuroscientific findings and have conducted studies to identify brain compatible strategies that teachers can employ. Working with the natural function of the brain can increase: (a) motivation, (b) transfer of knowledge, and (c) retention of information. Teachers can and should use this paradigm to promote successful learning.

Statement of the Problem

In the high school mathematics curriculum, the basics of trigonometry is included, which can be a difficult but potentially rewarding subject for students. Although the basic concept of trigonometry is simply a ratio of the sides of a right triangle, it is difficult for some students to transfer their knowledge of ratios to the concept of trigonometry. Educators can employ strategies based on the findings from brain research along with the research based knowledge of adolescent learning and self-efficacy to help their students to not only grasp the basics of trigonometry, but to go further and apply this knowledge to real world problems. Therefore, there is a need to incorporate the current research based information into a trigonometry unit to both increase students' motivation to learn this concept and their understanding of the concept.

Purpose of the Project

The purpose of this project will be to develop a trigonometry unit for the educators of high school students based on the findings from the current research on: (a) brain physiology, (b) adolescent learning, and (c) self-efficacy. With the use of research based methods, educators can help students develop a stronger foundation for the basic concepts of trigonometry. The use of hands-on projects, incorporated into the unit, will advance students' understanding and provide them with a sense of the real world applications of trigonometry. Also, this trigonometry unit will incorporate the current standards set forth by the Colorado Department of Education (CDE, 2005) and the National Council of Teachers of Mathematics (NCTM; Ferrini-Mundy et al., 2000).

Chapter Summary

Effective teaching leads to successful student learning. The intent of this project is to equip the teachers of high school geometry with a unit on trigonometry that uses strategies that work with the natural brain process to promote successful learning. The trigonometry unit will serve as a sample unit in which brain compatible teaching strategies have been applied. Teachers can transfer the strategies discussed in this unit to other units and subjects that they teach to increase their effectiveness.

In Chapter 2, the literature on brain research, adolescent learning, and self-efficacy is reviewed. Also, the standards for mathematics teaching are addressed. In Chapter 3, the procedure for the development of a trigonometry unit that is based on brain compatible teaching strategies is presented.

Chapter 2

REVIEW OF LITERATURE

Learning takes place in the brain; therefore, an understanding of the way the brain functions is an essential aspect of teaching. There has been an abundance of information accumulated in the last two decades by neuroscientists in the field of brain research. The use of this new information may transform, legitimatize, or debunk current trends in education. In the examination of teaching methods in light of the new research, some familiar teaching strategies have been found to be brain compatible while others have been shown to be detrimental (Smilkstein, 2003). Paradigms for brain compatible learning include new strategies to improve successful learning for all students. All facets of learning are linked to the brain and how it functions.

Educators should take advantage of the information about the brain that neuroscientists have provided for them to improve student learning as well as the lifetime prospects of their students. Therefore, the purpose of this project will be to develop a trigonometry unit that incorporates brain compatible strategies. This review of the literature will include: (a) the physiology of the brain, (b) brain based and natural learning strategies, (c) topics of specific importance to educators of adolescents, and (d) the Model Content Standards for Mathematics (Colorado Department of Education [CDE], 2005).

History of Brain Research

The study and discussion of thought started with the well known philosophers such as Plato, Aristotle, and Socrates (Marzano, 1988). In the mid 19th Century, psychologists began to study the mind with use of a scientific approach. In the late 19th and early 20th Centuries, learning theories were developed "as rapidly industrializing nations created systems for the masses" (Abbott & Ryan, 1999, p. 68). These learning theories were "behaviorist: people expected rewards to do tasks; their brains were blank sheets awaiting instruction; and intelligence was innate and largely inherited" (p. 68). A model for schools emerged that

> had a great deal in common with a model car--a car has a clear purpose; it has distinct and identifiable parts that make it run; and it's parts can be quantifiably assessed. When parts break, they are fixed; and low performance can be enhanced by using better gasoline or higher-quality oil. (Caine & Caine, 1997, p. 12)

Many learning theories have been developed over the years and used in the car model of schools. In the 1960s, the first learning theory that involved the biology of the brain emerged. In the information systems theory, "the central nervous system, the neurophysiology of the brain, and the electro-chemical discharges utilizing the high speed computer as a model of functioning" (White, 1996, p. 2) were explored. Since then, members of the medical and teaching professions have continued to gain vast amounts of information on the biological workings of the human brain.

In July, 1989, due to the tremendous amount of new information about the brain, "President Bush officially proclaimed the 1990's the 'decade of the brain' " (Wolfe & Brandt, 1998, p. 8). In the mid 1990s, some teachers started to use the new brain based or brain compatible learning techniques in their classrooms. Many of the so called new techniques had been used in the past but now there was a biological basis for why they worked. Brain researcher, Sylwester in his conversation with Brandt (1997), described teachers as brain researchers:

> if you're a teacher, you're dealing every day with about 100 pounds of brain tissue floating several feet above the classroom floor. Over a 20- or 30-year career, watching how those brains react, what they like to do, what they do easily and what they do with great difficulty, you're going to try to adapt your procedures to what works with brains. So, at that level, teachers have always been brain researchers. (p. 17)

The Brain

The brain is the hardest working organ in the body, it contributes less than 3% to the weight of the body, yet it uses 20% of its energy. When the brain works hard to solve complex problems, even more energy is consumed. The brain needs good nutrition and the right amounts of water and oxygen to function well (Smilkstein, 2003).

The human brain is composed of two types of cells (Jensen, 1998). Of the cells in the human brain, 90% are glia cells, and the remaining 10% are neurons. Glia cells are used by the brain: (a) to transport nutrients, (b) to regulate the immune system, and (c) to form the blood/brain barrier. Neurons are used for information processing and for the conversion of chemical and electrical signals back and forth (Sylwester, 1995).

In the neurons, energy flows down the axon to the synapse as shown in Figure 1. The cell will fire if there is enough combined energy that arrives from all of the dendrites. In the brain, many neurons fire simultaneously (Sylwester, 1995).

Learning requires groups of neurons (Jensen, 1998). When learning takes place, the axon splits into branches. When there are more connections, the brain is more efficient. Also, the brain becomes more efficient when myelin forms around well used axons (Sylwester, 1995; see Figure 2).

Figure 1. Direction of impulse through a neuron (Introductory Psychology Image Bank, 2007).

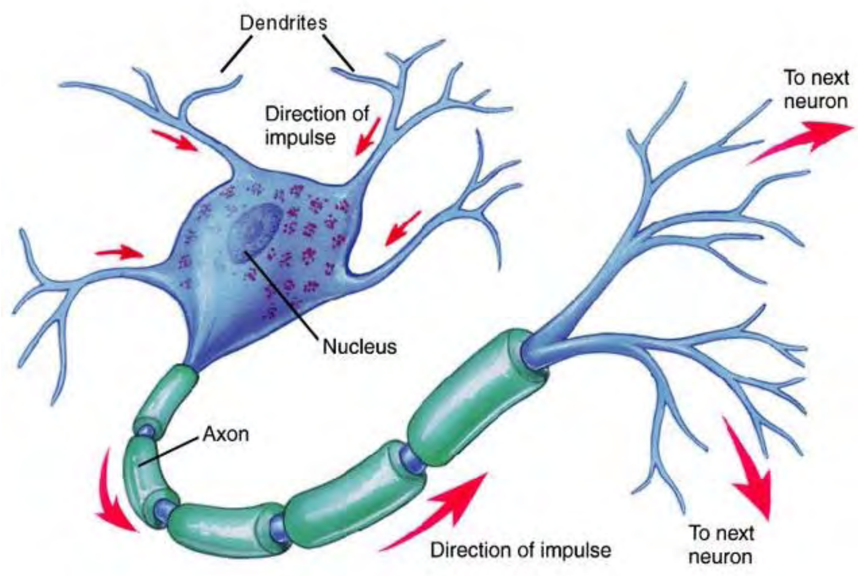
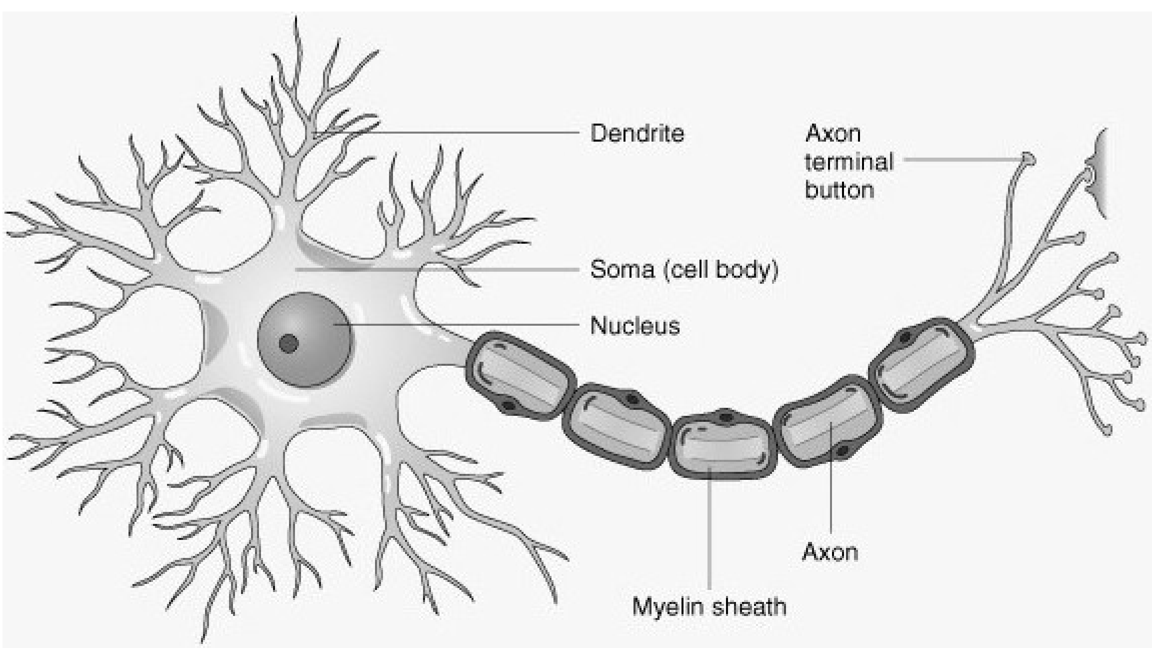


Figure 2. Diagram of a Neuron: Myelin sheath (Diagram of a Neuron, 2007).

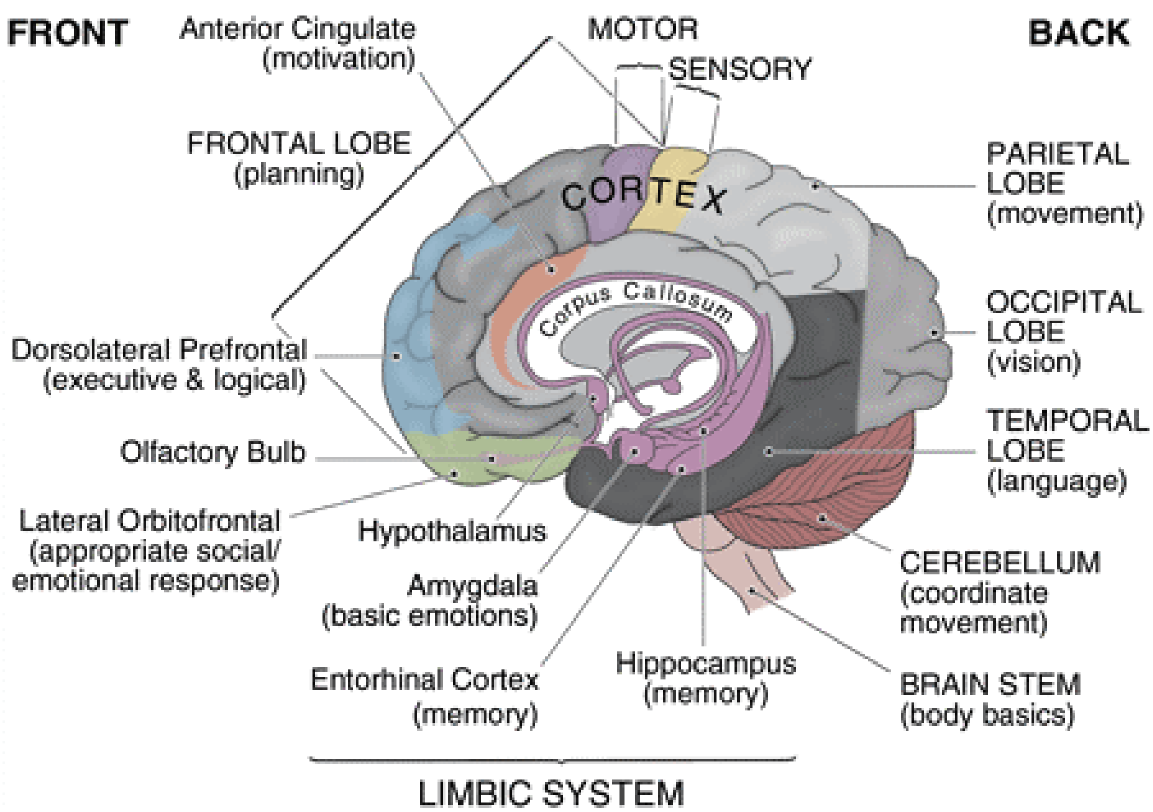


In addition, brain function is affected by hormones, such as endorphin, which reduce pain and increase euphoria (Caine & Caine, 1997). Pheromones regulate sexual behavior, levels of comfort, and self-confidence.

The terms that scientists use for the areas of the brain are shown in Figure 3 (Jensen, 1998). “Studies of human brains by neuroscientists have shown that different areas (lobes) of the cerebral cortex have separate functions” (Wolfe, 2001, p. 32).

1. Frontal Lobe: judgment, creativity, problem solving, planning
2. Parietal Lobe: processing higher sensory and language functions
3. Temporal Lobes: hearing, memory, meaning, language
4. Occipital Lobe: vision
5. Middle of the Brain (Lymbic System): emotions, sleep attention, body regulation, hormones, sexuality, smell, production of most of the brain’s chemical. (Jensen, 1998, pp. 8-10)

Figure 3. Diagram of the Brain: Terms used to describe the parts of the brain (Diagram of the Brain, 2007).



The scientific terms and brain functions listed will be referenced throughout this review of literature.

Tools Used for Brain Research

The relatively recent collection of knowledge about the human brain has been available due to the development and use of computer technology and brain imaging machines. Sylwester (1995) stated that "using imaging machines, researchers need only a few hours to gather from the brain the same type of data that formerly took 20 years of inferential laboratory work with nonhuman primates" (p. 12).

The equipment that is used in brain research is listed below (Jensen, 1998, see Appendix A for a more detailed description).

1. CAT scan; Computerized Axial Tomography
2. MRI; Magnetic Resonance Imaging
3. NMRI; Nuclear Magnetic Resonance Imagery
4. Spectrometers
5. EEG; Electro-encephalogram
6. SQUID; Superconducting Quantum Interference Device
7. BEAM; Brain Electrical Activity Mapping
8. PET; Positron Emission Tomography. (pp 2-4)

In addition to brain imaging machines, information is gathered by neurological pathologists when they perform autopsies (1998). Also, laboratory experiments with animals such as rats, dogs, cats, slugs, and apes provide information about the brain (Sylwester, 1995).

Brain Based or Brain Compatible Learning

One definition of brain based learning is as follows: "brain based learning involves acknowledging the brain's rules of meaningful learning and organizing teaching with those rules in mind" (Caine & Caine, 1991, p. 4). However, there is not complete agreement as to what the brain rules for meaningful learning consist of, and how teaching should be organized to use these rules. Caine and Caine (1997), a professor of education and a learning consultant, respectively, have compiled the following list of principles for brain/mind learning.

Principle 1:	The brain is a complex adaptive system.
Principle 2:	The brain is a social brain.
Principle 3:	The search for meaning is innate.
Principle 4:	The search for meaning occurs through patterning.
Principle 5:	Emotions are critical to patterning.
Principle 6:	Every brain simultaneously perceives and creates parts and wholes.
Principle 7:	Learning involves both focused attention and peripheral perception.
Principle 8:	Learning always involves conscious and unconscious processes.
Principle 9:	We have at least two ways of organizing memory.
Principle 10:	Learning is developmental.
Principle 11:	Complex learning is enhanced by challenge and inhibited threat.
Principle 12:	Every brain is uniquely organized. (p. 19)

Caine and Caine (1997) sought to combine these principles with traditional approaches to instruction. Their plan was implemented in two schools, an elementary school and a middle school, over a period of 4 years. According to Caine and Caine, they "wanted teachers to change from an almost universal belief in an 'information-delivery' approach to one that was flexible, creative, and open to students' search for meaning" (p. 241).

In the Caine and Caine (1997) study, three models of instructional approach were described. Each model has five aspects: "1) The objectives of instruction, 2) the teacher's use of time, 3) sources for curriculum and instruction, 4) How teachers define and deal with discipline, and 5) how teachers approach assessment" (p. 216).

Approach 1 is teacher controlled and rigid (Caine & Caine, 1997). The focus is on students' acquisition of specific facts and skills, under a strict time frame, and curricular sources are from texts. There are specific procedures for discipline, and students must exactly replicate the material which was presented.

Approach 2 is teacher controlled as well, but more complex and flexible; "it can incorporate powerful and engaging experiences, and teaching is often with an eye to create meaning" (Caine & Caine, 1997, p. 218). The focus is on a specific set of outcomes where: (a) the understanding of concepts is emphasized, (b) the time frame is flexible, (c) instruction includes a variety of sources, (d) discipline relates to student cooperation, and (e) several different performance evaluations are utilized.

Approach 3 is learner centered and nontraditional. Caine and Caine (1997) stated that "it is the type of teaching [they] had envisioned as brain based" (p. 219). In Approach 3, curriculum is based on knowledge that students can use in everyday life, the time frame is linked to the learner's needs, there are multiple sources for instruction which reflect student interests, discipline is nontraditional, and assessments are centered on students' demonstration of understanding and application of the knowledge to real world situations.

Caine and Caine's (1997) goal for teachers was to change their instruction to Approach 2, although they shared all three approaches with the teachers. They found that only some of the teachers had the ability to use instructional Approach 3. Caine and Caine felt that they were moderately successful. They found that it was difficult to change a school in its entirety, and the shift to "brain-based teaching required teacher transformation in some demanding ways" (p. 241).

Caine and Caine (1997) reported that, the teachers' ability to implement the instructional approaches was found to be associated with their perceptions and beliefs. Also, they found that teachers' perceptual orientations had four dimensions.

1. A sense of self-efficacy grounded in authenticity
2. The ability to build relationships that facilitate self-organization.
3. The ability to see connections between subjects, discipline, and life.
4. The capacity to engage in self-reflection to grow and adapt. (p. 221)

In Addition, Caine and Caine (1997) discussed another major issue which they linked with brain research. People in the industrialized world are leaving the industrial age and entering the age of information where more information is available to more people. The Caines feel that "the basic model we have of how the world works is itself being called into question" (p. 12). They mentioned that the combination, of living in the information age and the increasing knowledge of how the brain works, will cause a change in societal beliefs in regard to education, from the traditional beliefs that:

1. only experts create knowledge,
2. teachers deliver knowledge in the form of information, and
3. children are graded on how much of the information they have stored.

to a different set of beliefs :

1. dynamical knowledge requires individual meaning making based on multiple sources of information,
2. the role of educators is to facilitate the making of dynamical knowledge, and
3. dynamical knowledge is revealed through real world performance. (p. 258)

Not all of the interpreters of brain research have taken this information to the extreme that Caine and Caine (1997) did. "Researchers especially caution educators to resist the temptation to adopt policies on the basis of a single study or to use neuroscience as a promotional tool for a pet program" (Wolfe & Brandt, 1998, p. 8). Wolfe and Brandt supported a moderate approach for the interpretation of brain research. They mentioned that: (a) educators are best suited to determine how brain research is interpreted for classroom use, and (b) the recent brain research confirms some of the methods that some teachers use and indicates other areas where changes should be made. Informed teachers can decide whether they are working with their students' brains or against the brains of their students.

According to Wolfe and Brandt (1998), "the environment in which the brain operates determines to a large degree the functioning ability of that brain" (p. 9). They provided four suggestions to help teachers determine whether their classroom environments are brain compatible.

1. An enriched environment gives students the opportunity to make sense, make meaning out of what they learn.
2. In an enriched environment, multiple aspects of development are addressed simultaneously.
3. The brain is essentially curious and constantly seeks connections between the new and the known.
4. The brain is innately social and collaborative. Learning is enhanced when students have opportunity to discuss their thinking out loud.

Also, one must keep in mind that the human brain is interconnected with a person (D'Arcangelo, 1998). One cannot treat a student simply as a brain. Constantly, all of a person's senses affect the personal environment of the brain. In addition, emotions are strongly linked to brain function. "Our emotional system drives our attention, which drives learning and memory and everything else we do" (p. 25).

Windows of Opportunity and Brain Plasticity

There are certain *windows of opportunity* for learning (Wolfe & Brandt, 1998). "The best time to master a skill associated with a system is just when the new system is coming on line in your brain" (p. 23). Once this period of time passes, learning these skills may be difficult or nearly impossible. Vision and language development have the most clearly defined periods. These periods end at a young age and are linked to skills that increase the ability for a young human to survive.

However, all learning does not take place when one is young (Wolfe & Brandt, 1998). On the contrary, the plasticity of the brain allows people to learn new things at any age (Caine & Caine, 1991). Physiological and psychological changes in the brain continue throughout one's lifetime. Life long learners continue to keep their brains active and keep neuron connections strong. "With enrichment we grow those dendrites; with impoverishment, we lose them at any age" (p. 23).

From 1996-1998, Swedish neurologist Eriksson (as cited in Begley, 2007) examined the brains of deceased cancer patients who had been treated with BrdU, luminous green molecules that attach to newborn cells. He found evidence of neurogenesis, the physical process of cells being born and developing, in the human brain.

> The discovery overturned generations of conventional wisdom in neuroscience. The human brain is not limited to the neurons it is born with, or even to the neurons that fill it after the explosion of brain development in early childhood. New neurons are born well into the eighth decade of life. They migrate to structures where they weave themselves into existing brain circuitry and perhaps form the basis of new circuitry. (p. 65)

In further research conducted on animals, it was found that physical activity generates new brain cells and an enriched environment “affects the rate and the number of cells that survive and integrate into the circuitry” (Gage, 2004, as quoted in Begley, p. 66).

The Natural Learning Process

Human beings are born with a natural motivation to learn, and students need to be given the opportunity to learn naturally in school (Smilkstein, 2003). The provision of brain compatible environments and activities allow students to be motivated and learn naturally.

Smilkstein (2003) conducted research on her theory of the natural human learning process (NHLP) with more than 5,000 individuals, and she has identified the common experiences of learning something new. The findings suggest that learning is a process of 4-6 stages. Smilkstein summarized the stages as follows.

> Stage 1: Motivation/Responding to stimulus in the environment: watched, observed, had to, interest, desire, curiosity.
>
> Stage 2: Beginning Practice/Doing it: practice, practice, practice, trial and error, ask questions, consult others, basics, make mistakes, lessons, some success.
>
> Stage 3: Advanced Practice/Increase of skill and confidence: practice, practice, practice, trial and error, some control, reading, encouragement, experiment, tried new ways, positive attitude, enjoyment, lessons, feedback, confidence, having some success, start sharing.
>
> Stage 4: Skillfulness/Creativity: Practice, doing it one’s own way, feeling good about yourself, positive reinforcement, sharing knowledge, success, confidence.
>
> Stage 5: Refinement/Further improvement: learning new methods, becoming second nature, continuing to develop, different from anyone else, creativity, independence, validation by others, ownership, habit, teaching.
>
> Stage 6: Mastery/Application: greater challenges, teaching it, continuing improvement or dropping it, feeds into other interests, getting good and better and better, going to higher levels. (p. 49)

According to Smilkstein (2003), the brain has innate resources for learning, and a sequence of rules is followed when learning takes place. The innate resources of the brain are:

> The brain has a natural learning process; the brain has an innate sense of logic; the brain is an innate pattern seeker; the brain is an innate problem solver; the brain is innately imaginative and creative (can see new ways); the brain is innately motivated to learn. (p. 71)

Smilkstein identified five rules that the brain follows during the learning process. First, new learning is connected to something that is already known because dendrites, synapses, and neural networks can only grow from what has already been formed. Second, people learn and remember by practice because dendrites, synapses, and neural networks are formed by active, personal experiences. Third, learning requires interactive feedback with other individuals because dendrites, synapses, and neural networks form during stimulating experiences. Fourth, when a skill is not used, the ability to perform it is diminished over time because the brain prunes unused neural networks in an effort to become more efficient. Fifth, "emotions, thinking and learning are inextricably bound together" (p. 86) because emotions produce chemicals that affect the brain which affects learning.

Both students and teachers can employ specific strategies to facilitate successful learning. Smilkstein's (2003) findings demonstrated that teaching students about metacognition and the NHLP results in self-empowerment and confidence. Students' ability to self-evaluate in regard to metacognition and the NHLP leads to their being able to take responsibility for their own learning.

According to Smilkstein (2003), teachers can help students to learn if they remove some of the barriers that impede the NHLP. Notable barriers to learning occur when: (a) students are overburdened with large quantities of factual information, (b) providing students with inadequate involvement to facilitate the connection of the new material to previously learned information, and (c) too many passive learning experiences with insufficient time for students to build their own knowledge.

Smilkstein (2003) maintained that educators must facilitate and stimulate the brain structure growth of their students. This leads to successful learning. Every brain has a unique neural network which results in differences in the acquisition of knowledge such as: (a) learning preferences, (b) ability, (c) strengths, (d) intelligences, and (e) so on. In an effort to accommodate all learners, educators have tried to implement a wide array of instructional strategies. However, it has been found that there is “no one type of curriculum or pedagogy that makes it possible for all students to learn successfully” (p. 86). Instead, Smilkstein recommended that teachers should develop lessons based on the universal process by which human beings learn, the NHLP.

The knowledge of how the brain learns has been translated into pedagogy for teaching courses. Smilkstein (2003) cited Boylan (2002) who identified a natural learning pedagogical approach with four major instructional practices: “feedback (both instructor and peer), active learning, self-monitoring, and assessment” (p. 152). With the use of Boylan’s natural learning pedagogical approach, Smilkstein conceptualized a three step process which is followed by assessment. The three step process is: (a) individual work, (b) small group activity/interactivity, and (c) whole group feedback.

First, the use of individual work allows the learners to make connections to their unique set of neural connections via prior knowledge (Smilkstein, 2003). Second, small group activity encourages feedback and practice which causes knowledge structures to grow, that is, learning. Third, the use of whole group session allows students to contribute their own individual ideas and listen to the contributions of others; this results in the development of more neural connections in the individual.

According to Smilkstein (2003), during the unit, the teacher uses a formative assessment to check student understanding and asks them to rate their understanding on a scale of 1-6. Assessment at the end of the unit incorporates both student and teacher evaluations. Students evaluate the teacher in regard to: (a) what helped them learn, (b) what did not work, and (c) suggestions for improvement. Then the teacher develops a summative assessment. Students are involved in the development of the summative assessment and are asked to produce review questions which are combined with information from the teacher. Time is allowed for study. The questions are reviewed and answered during class. Finally, the examination is administered to the students.

Some of the traditional classroom learning experiences and environments are not compatible with the natural learning process, for example, memorization of the right answers or learning only one way to correctly perform a task (Smilkstein, 2003). Direct transfer of knowledge from the teacher to the student does not always lead to students being able to transfer the learned information to new situations. Some students learn successfully in traditional classrooms, while other students, who may learn differently, experience failure. Levine (2002) believes that all students possess identifiable strengths and weaknesses. "We can cultivate [students'] minds by addressing the weaknesses and strengthening the strengths" (p. 15).

Neurodevelopmental Systems

Levine (2002) developed a model of learning based on 3 decades of: (a) clinical observations, (b) collaboration with schools worldwide, (c) neuroscientific literature, and (d) his research. According to Levine, "the most basic instrument for learning is something called a neurodevelopmental function" (p. 28). He groups the functions into 8 neurodevelopmental systems: (a) the attention control system, (b) the memory system, (c) the language system, (d) the spatial ordering system, (e) the sequential ordering system, (f) the motor system, (g) the higher thinking system, and (h) the social thinking system. Educators are, in part, responsible for the healthy growth of these systems and should "keep an eye on the progress in each system, promptly detecting and dealing with any important impairments or signs of delayed development" (p. 31).

According to Levine (2002), neurodevelopmental dysfunctions of the brain sometimes, but not always, leads to poor academic performance and a downward spiral toward failure. "Tragic results are seen when we misconstrue and possibly even misuse a child's kind of mind! And this happens all the time" (p. 13). Teachers should be knowledgeable about the neurodevelopmental systems in order to meet the educational needs of students. The neurodevelopmental systems were applied by Levine into a list of the aims of education (see Appendix B). The years of formal education should both enrich students and allow them to see possibilities for themselves. Also, Levine developed a list of the main characteristics of a school for all kinds of minds (see Appendix C). The awareness of neurodevelopmental variation has "moral and ethical implications. . . . [and] we must make a firm social and political commitment to neurodevelopmental plurism" (p. 335).

The Adolescent Brain

Adolescence is defined as the period in human development between puberty and adulthood (Walsh, 2004, p. 15). The span of adolescence has increased; usually, the onset of puberty in the 19th Century was at the average age of 17, today, the average age is 12. Theories on the reasons of earlier puberty include: (a) better nutrition, (b) childhood obesity, (c) sexual images on television, (d) food additives, (e) processed food, and (f) growth hormones fed to animals. Adolescence ends when adult roles and responsibility are taken on by the individual. Today, an individual may be 25 years of age before these roles are assumed due to postsecondary school or training. From an educator's point of view, adolescence spans the years that a student is in middle school, high school, and beyond.

Although the adolescent brain is the same size as an adult's, "scientists now know that the adolescent brain is not a finished product but a work in progress" (Walsh, 2004, p. 17). There are five processes of brain development: "(a) use it or lose it, (b) blossoming and pruning, (c) the window of opportunity, (d) the window of sensitivity, and (e) myelination" (p. 32).

According to Walsh (2004), there are many facets to adolescent brain development. The frontal lobes, which are involved in critical thinking and problem solving, go through a growth spurt between the ages of 10-12. The prefrontal cortex, the area of the brain that controls impulses and allows an individual to think ahead to the consequences of an action, blossoms with an overproduction of brain cell branches which peaks at about the age of 12. Then the pruning process continues throughout adolescence. The ability to begin to think abstractly is acquired between the ages of 11-16. Also, at approximately age 16, the temporal lobes which control emotion and language, undergo pruning. “The myelination process in certain parts of the teen brain actually increases by 100 percent from the beginning of adolescence to the end” (p. 37). During the 20s, additional unused synapses are eliminated which allows the remaining networks to be more efficient.

In addition to development, the adolescent brain is affected by surges of hormones (Walsh, 2004). There are three growth hormones: (a) testosterone, (b) estrogen, and (c) progesterone. In boys, the testosterone affects the amygdala, the part of the brain that controls the fight-or-flight response, which may lead to negative behaviors. In girls, estrogen and progesterone affect the hippocampus which controls memory and, potentially, may be beneficial to the learning process.

Also, the hormone fluctuations in adolescents affect the neurotransmitters in the brain (Walsh, 2004). They create extreme impulses. Neurotransmitters are the brain chemicals that carry nerve impulses between neurons. Serotonin is a mood stabilizer and helps one to feel relaxed and confident, dopamine is called the feel good chemical, and norepinephrine causes the fight-or-flight feeling. The surge of chemicals in boys can make them become angry or aggressive. In girls, the surge creates an amplification of emotions. The adolescent brain must contend with the surges of emotion while the brain is still developing its ability to control impulses and to think ahead to the consequences of an action.

The new emotions that emerge in adolescence are: "(a) sexual desires, (b) jealousy, and (c) territoriality" (Walsh, 2004, p. 218). The adolescent must learn how to react to these new emotions while he or she experiences the increased intensity of more familiar emotions such as anger and sadness.

Walsh (2004) stated that "Deborah Yurgelun-Todd, a researcher at McLean Hospital outside Boston, and other brain scientists have discovered that the adolescent brain interprets emotional expressions differently than an adult's brain" (p. 77). The photographs used in the study showed expressions of: (a) anger, (b) sadness, (c) surprise, and (d) fear. The results showed that the adult participants interpreted emotional expression and could distinguish subtle differences mainly through the use of the prefrontal cortex which employs reasoning and rational thought. In the adolescent participants, the amygdala was used. However, the amygdala, according to Walsh, "reacts first and asks questions later" (p. 79). It is the part of the brain associated with: (a) fear, (b) anger, and (c) emotion. "Adults could correctly identify different emotional states in the pictures, but adolescents often mistook fear or surprise for anger. As scientists studied the data more closely, they found that adolescents frequently misread emotional signals" (p. 78).

Self-Efficacy

Self-efficacy is the belief in one's ability to successfully perform a task (Margolis & McCabe, 2006). Unlike self-concept, which is based on a person's comparison of how well he or she can perform in reference to others, self-efficacy involves a personal judgment on whether one has the ability to master a specific task or, in other words, attain a goal successfully. According to Bandura and Locke (2003),

> Among the mechanisms of human agency, none is more central or pervasive than beliefs of personal efficacy. Whatever other factors serve as guides and motivators, they are rooted in the core belief that one has the power to produce desired effects; otherwise one has little incentive to act or persevere in the face of difficulties. (p. 87)

The characteristics of people with high perceived self-efficacy include: (a) perceive difficult tasks as challenges rather than threats that need to be avoided; (b) maintain strong commitments; (c) set high goals, (d) concentrate on the task rather than on themselves; (e) blame failures on lack of preparation, effort, or skill; (f) redouble effort in the face of obstacles instead of giving up; and (g) recover confidence after setbacks.

According to Bandura and Locke (2003), efficacy beliefs are a notable factor in one's: (a) motivation, (b) perseverance, and (c) performance. In a study conducted by Cervone and Peake (1986, as cited in Bandura & Locke), perceived self-efficacy was altered by the introduction of an arbitrary reference point. The participants were asked to rate their efficacy with use of arbitrarily high and low starting numbers to raise and lower perceived self-efficacy. "The higher the. . . perceived self-efficacy was, the longer individuals persevered on difficult and unsolvable problems before they quit" (2003, p. 88).

In other studies conducted by Bandura and Adams (1997) and Bouffard-Bouchard (1990, both cited in Bandura & Locke, 2003), efficacy beliefs were raised by the use of visualization and by normative comparison. Use of these means to raise or lower perceived self-efficacy has shown that efficacy can regulate a child's performance and mastery of academic tasks. The child's current performance is not based on a reflection of past performance, but on perceived self-efficacy. Self-efficacy beliefs can be influenced by past performance, but they are not determined by past performance.

According to Margolis and McCabe (2006), students with high efficacy beliefs have the motivation, confidence, and drive to achieve academically. Low self-efficacy in students causes motivation problems and can lead to a self-fulfilling prophecy of failure and learned helplessness. Low achieving students "bombard themselves with dysfunctional attributions, erroneously convincing themselves that they lack needed abilities" (p. 225). Margolis and McCabe stated:

> to increase the likelihood that attribution retraining will succeed, teachers should stress the following (Alderman, 2004; Linnenbrink & Pintrich, 2002; Mushinski-Fulk & Mastropieri, 1990; Pintrich & Schunk, 2002; Ring & Reetz, 2000): success is due to controllable factors. These factors include strong, persistent effort. . . correct use of specific learning strategies. . . . Failure is due to inadequate, short-lived effort, half-hearted or incorrect use of specific learning strategies, . . . or inadequate information. . . . Failure is not due to permanent limitations. (p. 225)

Methods to Strengthen Self-Efficacy

As reported by Bandura (1997), efficacy beliefs can be altered in four main ways: (a) experience of success or mastery, (b) social modeling, (c) social persuasion, and (d) reduction of stress and depression. In the educational setting, strategies can be employed that reflect these alteration techniques. According to Margolis and McCabe (2006), students can experience success or mastery when they complete tasks of moderate challenge, slightly above the learners' current level of performance. These academic tasks should be completed with only moderate effort on the student's part. A task that requires too little effort will not give the student a sense of mastery, while a task that requires excessive effort will cause fatigue and be interpreted by the learner as a sign of personal inadequacy. Moderately challenging tasks should result in 80% or better correct responses for guided practice (e.g., class work) and 95% or more correct responses on independent work (e.g., homework).

Social modeling can be provided in the classroom through the provision of vicarious experiences (Margolis & McCabe, 2006). A student's self–efficacy can be strengthened by observation of peer models, in particular, models that share similarities such as: (a) age, (b) gender, (c) clothing, (d) social circles, (e) achievement levels, or (f) race. The peer models provide the observer with direct guidance by explanation of what they do and think while they complete the task.

Social persuasion is provided by the teacher's use of verbal persuasion (Margolis & McCabe, 2006). The teacher gives the student individual feedback and prompts, which lead to successful completion of the task. Also, the teacher needs to remain credible in the eyes of the student. The teacher must only tell the student that he or she will succeed on tasks that the teacher is sure the student has the ability to complete successfully. The teacher follows up and states what the learner did that produced success.

Reduction of stress and anxiety can be addressed by teaching the student how to relax and how to manage the irrational thoughts that bring about these physiological reactions (Margolis & McCabe, 2006). A student, who is unable to reduce anxiety through appropriate methods, may turn to escape behaviors, such as class disruption, in order to avoid the task and thereby alleviate stress.

Educators can strengthen the learner's self-efficacy with the use of several methods (Margolis & McCabe, 2006). First, students should over-learn strategies to attack a problem. Second, the student needs to be given a logical sequence of steps to use when he or she solves problems as well as guidelines for their appropriate use. Third, an abundance of guided and independent practice should be provided. Fourth, it should be emphasized that success is due to: (a) controllable factors, (b) strong persistent effort, and (c) correct use of strategies. Students need to be told when to use the strategy and when they have successfully learned it. The teacher needs to reinforce student effort, his or her use of the correct strategy, and provide encouragement and deserved praise. When a new task is to be undertaken, the teacher can emphasize that success is likely if the student makes the effort, and the teacher should point out how the new work resembles old work.

Initially, students can be motivated with the use of extrinsic reinforcers, which are phased out eventually (Margolis & McCabe, 2006). Other methods to initialize motivation are: (a) give the student a choice, (b) respond to student interests, (c) provide novelty, and (c) provide relevance.

In a study conducted by Caraway, Tucker, Reinke, and Hall (2003) of adolescents' level of school engagement, it was found that student self-efficacy and goal orientation were major components. According to Caraway et al., "engagement appears to be the cornerstone of academic achievement motivation" (p. 418). Their findings suggested that adolescents' confidence in their level of competence increases school engagement, which leads to increased academic achievement. The purpose of the strategies suggested for teachers were to increase positive feedback as well as to increase the frequency of feedback.

School Environment

In a study conducted by Anderson, Hattie, and Hamilton (2005), three different types of schools were compared to investigate the relationship between the school structure and academic motivation. The participants were categorized as either externals, who preferred high discipline and more structure, or internals, who had greater self-efficacy and internal locus of control. Their results showed that "the non-extreme school judged to have medium levels of structure, competitiveness, and cooperation was associated with higher levels of motivation and achievement than the other schools" (p. 532). The highly structured, most competitive school was found to have a negative effect "on motivation in locus of control internals" (p. 533).

Adolescent Goals

"Much of human behavior is goal directed" (Carroll, Durkin, Hattie, & Houghton, 1997, p. 441). As stated previously, goal orientation has been found to be a major factor in school engagement, which ultimately leads to school achievement (Caraway et al., 2003). The goal of educators should be to pass on their knowledge and skills to their students and help students to formulate their own goals. Formulation of personal goals has long term implications, and adolescence is a critical period for this formulation.

According to Carroll et al. (1997), an adolescent is concerned primarily with: (a) peer relationships, (b) social identity, (c) self-concept, (d) reputation, and (e) development of personal autonomy. These concerns are reflected in their formulation of goals in regard to: (a) social matters, (b) personal matters, (c) education, and (d) career. Educational goals are prominent in the minds of adolescents, and researchers (Nurmi, 1987, 1989a, 1989b, 1991a, 1991b; Salmela-Aro, et al., 1991; all cited in Carroll et al., 1997) have shown that finishing their education is one of the highest goals of an adolescent.

Delinquent, At-Risk, and Not At-Risk Youth

Although adolescents may have similar types of goals, the way that they rank the goals in regard to importance can vary. Carroll et al. (1997) compared the importance placed on the different types of goals by: (a) delinquent, (b) at-risk, and (c) not at-risk youth. Their findings supported previous research (Duda & Nicholls, 1992; Nicholls, 1984, 1989; Wentzel, 1989; all cited in Carroll et al.) which showed that "the goal-setting patterns of high-achieving students were congruent with those of their educational institution" (p. 448). Students in the not at risk group placed greater importance on educational and interpersonal goals (i.e., Academic Image), while both the delinquent and at-risk groups placed higher importance on delinquency and freedom autonomy goals (i.e., Social Image). The importance placed on goals for self-presentation, reputation, and career was found to be the same for: (a) delinquent, (b) at-risk, and (c) not at-risk youth.

Students' perception of ability level and their interpretations of classroom success and failure influence achievement goals. However, the findings from current research conducted by Urdan and Mestas (2006) suggested that it may be more multifaceted. In the study, students were asked to describe the reasons they pursued performance goals in their own words. Their findings suggested that motivation in the classroom may be affected by the variety of ways that students think about goals and by the multiple reasons they pursue these goals. According to Urdan and Mestas, "achievement goals may be more complex and multidimentional than often depicted in research" (p. 364).

Urdan and Mestas (2006) identified a goal that is not generally explored in goal theory research, that is, "the goal of not distinguishing oneself" (p. 362). An additional finding was that it was difficult for students to distinguish subtle differences in the researchers' questions. Students' responses on some questions reflected different goal items than were intended by the researchers. The implications for goal theory and research were: (a) there are a variety of reasons for the pursuit of performance goals, and (b) participants may interpret performance goal items in unexpected ways.

Perfectionists

Students who are perfectionists are part of the typical classroom body (Accordino, Accordino, & Slaney, 2000). These students have high standards for performance goals. However, their academic achievement can be affected by the degree of their perfectionism. Psychologists divide perfectionists into two types, normal and neurotic. According to Accordino et al., "individuals with normal perfectionism tend to derive pleasure from striving to meet challenging but attainable goals. . . . Neurotic perfectionists, on the other hand, are characterized as having high levels of anxiety and a strong fear of failure" (p. 536). Neurotic perfectionists feel they can never meet their own expectations. Minor mistakes seem catastrophic and result in high levels of discrepancy. Normal perfectionism results in greater achievement and motivation while neurotic perfectionism results in lowered achievement, lowered self-esteem, and it can eventually lead to depression. Accordino et al. stated that it would "be worthwhile to have knowledge of a student's perfectionist tendencies. . . [and school psychologists could teach] students how to set and achieve personal goals as well as giving students positive reinforcement for achieving those goals" (p. 543).

Efficacy and Goals

Although the pursuit of a college degree may not be appropriate for every student, "the economic consequences of not obtaining a college degree seem to be greater today and may likely be greater in the future if trends persist” (Trusty, 2000, p. 356). Students' self-efficacy is strongly influenced by their parents’ aspirations and efficacy. A longitudinal study was conducted by Trusty to investigate the stability of adolescents’ postsecondary educational expectations. The expectation, as eighth grade students (*N* = 2,265), to obtain a bachelor’s, degree was held by all participants included in the study. The participants were interviewed in eighth grade, and a follow-up questionnaire was completed 6 years later. Several variables were found to be predictive of stable expectations: (a) eighth grade mathematics achievement, (b) mothers’ early expectations, and (c) locus of control. Reading achievement, the availability of a computer in the home, and fathers’ expectations were not predictive of stable expectations.

In earlier studies (National Education Longitudinal Study, 1988; Hafner, Ingels, Schneider, & Stevenson, 1990; Mickelson, 1990; all cited in Trusty, 2000), researchers found a discrepancy between high educational aspirations and plans for preparation to meet the goal. The low socioeconomic status (SES) students were found to have unrealistic educational expectations due to parental pressure and overestimation of their abilities (Agnew & Jones, 1988, as cited in Trusty). African American and low SES students were found to have low achievement. In regard to ethnic groups, the findings supported those of previous studies of high achievers (Hanson, 1994; Trusty & Harris, 1999; both cited in Trusty) which “revealed that [Anglo Americans] were more likely to have lowered expectations than members of minority groups” (p. 358). Asian, Hispanic, and African American males had stable expectations. Hispanic and Anglo American females were likely to have lower expectations.

Students, who used the services of counselors and teachers, were less likely to feel that they could reach their educational goals, possibly because they were influenced to be more realistic (Trusty, 2000). Extracurricular school activity attendance was related to stable expectations. The findings suggested that higher levels of parent involvement were associated with the adolescents' positive perception of education and career. "Self-efficacy for long term educational achievement was significant [$p < .05$] in both female and male models" (p. 364).

Motivation

In the past, teachers have used external rewards such as candy or prizes in an attempt to motivate their students (Jensen, 1998). Originally, many teachers misinterpreted the implications from behaviorism research (e.g., the stimulus response rewards meant for simple physical actions) and used these rewards for complex problem solving tasks. Researchers (Amabile, 1989; Kohn, 1993; both cited in Jensen) have shown that external rewards are effective for some students and not for others. Also, students tend to want repeated rewards for the same behavior as well as increasingly valuable rewards. The effects of external rewards are small and short lived, and they have been shown to damage intrinsic motivation.

Motivation is a critical element of learning, and people have a natural motivation to learn; however, also, motivation is subject specific. Sometimes, students are sufficiently motivated to make it to school, but not motivated enough to be involved in classroom learning (Jensen, 1998). One cause of this phenomenon is past associations, which can produce a negative or apathetic state. Something about the teacher, classroom, or subject can cause the amygdala to release chemicals that create bad feelings. Another reason may be due to environmental factors such as: (a) hunger, (b) classroom environment, or (c) teaching styles. Finally, the student may lack the goals and beliefs, in relation to the subject or teacher, that create positive thinking. When positive thinking occurs, pleasure chemicals such as dopamine, endorphins, and opiates are released in the brain.

Chemicals in the brain regulate stress and pain as well as produce a natural high (Jensen, 1998). The hypothalamic system releases pleasure producing chemicals that makes one feel good. Jensen reported that the internal reward system of the brain works "like a thermostat or personal trainer, your limbic system ordinarily rewards cerebral learning with good feelings on a daily basis" (p. 65).

Unlike external rewards, internal rewards are beneficial to motivation (Jensen, 1998). Several factors are present with intrinsic motivation such as: "compelling goals, positive beliefs, and productive emotions" (p. 67). Although neuroscientific researchers have not determined the biological functions involved with goals and beliefs, the functions involved with emotion have been identified. The brain chemicals, noripinephrine and dopamine, have been observed in increased levels during mild cognitive motivation, also, increased levels of the hormones vasopressin and adrenaline, are associated with stronger, more active motivation.

Jensen (1998) suggested five strategies that teachers can use to increase the release of the chemicals involved with intrinsic motivation in their student's brains.

1. Eliminate threat
2. Goal-setting
3. Positively influence students' beliefs about themselves and the learning
4. Manage student emotions and teach them how to manage their emotions
5. Give feedback. (p. 69)

National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics (NCTM; Ferrini-Mundy et al., 2000) is “an international professional organization committed to excellence in mathematics teaching and learning for all students” (p. ix). The following goals for mathematics instruction of students in Grades 9-12 are quoted from the resource guide, *Principles and Standards for School Mathematics*. This author has included only the standards which are related to this current project. The standards for Algebra, Data Analysis and Probability, and Reasoning and Proof have been omitted.

> Number and Operations: Students should compute fluently with real numbers and should have some basic proficiency with vectors. . . in solving problems, using technology as appropriate. . . . [They should be able to] judge the reasonableness of numerical computations and their results. (p. 32)
>
> Geometry: High school students should use Cartesian coordinates as a means both to solve problems and to prove their results. . . . High school students should be able to visualize and draw. . . cross sections of [block] structures and a range of geometric solids. (pp. 42-43)
>
> Measurement: Students should come to recognize the need to report an appropriate number of significant digits when computing with measurement. . . . In high school, as students use formulas in solving problems, they should. . . [be able] to organize their conversions and computations using unit analysis. . . . High school students should study more sophisticated aspects of scaling. (pp. 44-47)
>
> Problem Solving: Instruction should enable students to. . . build new mathematical knowledge through problem solving, solve problems that arise in mathematics and other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving. . . . By high school, students should have access to a wide variety of strategies, be able to decide which one to use, and be able to adapt or invent strategies. (p. 52)
>
> Communication: Instruction should enable students to. . . organize and consolidate their mathematical thinking through communication, communicate . . . coherently;. . . analyze and evaluate the mathematical thinking and strategies

> of others, and use the language of mathematics to express mathematical ideas precisely. (p. 60)
>
> Connections: Instruction should enable students to. . . recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics. . . . In grades 9-12 students should be confidently using mathematics to explain complex applications in the outside world. (p. 64)
>
> Representation: Instruction should enable students to. . . create and use representation to organize, record, and communicate mathematical ideas; select apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena. (p. 67)

Colorado Model Content Standards

In order to develop the mathematical literacy of K-12 students in Colorado, the Colorado Model Content Standards(CDE, 2005) were developed. The Model Content Standards were adapted from the NCTM Standards by a group of Colorado mathematics teachers. The Colorado Model Content Standards serve as a guide for school district staff to utilize in their effort to define district level standards. The Colorado State Board of Education adopted the standards on June 8, 1995. The six goals that serve as the framework of the Colorado Model Content Standards for Mathematics are as follows:

Standard 1	Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.
Standard 2	Students use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.
Standard 3	Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.
Standard 4	Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.

Standard 5	Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.
Standard 6	Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem solving situations and communicate the reasoning used in solving these problems. (p. 2)

With use of the six goals as a framework, the developers divided the Colorado Model Content Standards into three grade level categories and described them in more detail in the Benchmarks for each standard (see Appendix D for a copy of the detailed information for Grades 9-12 Colorado Model Content Standards).

Chapter Summary

Brain function research, combined with teachers' knowledge of students and how they learn best, could lead to exciting accomplishments in the field of education. Teachers would be wise to learn about the functioning of the human brain and keep informed about current research.

Teachers should learn about the human brain but, as they learn, there is a need for caution. This is a common warning throughout the literature written for teachers about brain research. Sylwester (1995) summarized it in the following statement:

> Can a profession whose charge is defined by the development of an effective and efficient human brain continue to remain uninformed about that brain? If we do remain uninformed, we will become vulnerable to the pseudoscientific fads, generalizations, and programs that will surely arise from the pool of brain research. We've already demonstrated our vulnerability with the educational spillover of the split brain research: the right brain/left brain books, workshops and curricular programs whose recommendations often went far beyond the research findings. (p. 6)

Knowledge about the functioning of the brain will only increase over time. Many students could benefit from their teachers' competence in this area. Teachers are in the best position to decide how information from brain research can be used in a classroom setting. In Chapter 3, the procedures used to develop a trigonometry unit in line with the brain compatible paradigm will be presented.

Chapter 3

METHOD

The purpose of this project was to develop a trigonometry unit for educators of high school adolescents, based on current research on brain compatible learning, in order to increase students' motivation to learn successfully. The level of mathematical competence a high school graduate possesses can either facilitate, or be a detriment, to future career options available for him or her to pursue. Also, in the typical high school classroom, teachers instruct a diverse population of students, in regard to: (a) academic ability, (b) social values, and (c) individual personalities and needs. Therefore, mathematics instructors, will be interested in teaching methods which can help all learners be successful. Educators who adopt a brain compatible paradigm will be able to not only attain a goal of successful mathematical learners, but also improve society by their promotion of the higher level thinking abilities of all their students.

Targeted Audience

This project was designed for application with adolescent geometry students in Grades 9-12, although the strategies and activities described could be adapted for students in other grade levels and subjects. Educators, who are interested in teaching strategies that work with the natural learning function of the brain, will be interested in this project.

Goals of the Applied Project

The project provided geometry teachers with a trigonometry unit which was based on brain compatible teaching methods and geared toward adolescent learning. First, this author provided an overview of the classroom environment that promotes brain compatible learning. Second, a unit on trigonometry was presented which used research based strategies for brain compatible learning and incorporated the Colorado Model Content Standards for Mathematics (Colorado Department of Education, 2005). This unit served as an example for teachers who want to provide students with an effective learning experience which is brain compatible and based on research.

Procedures

A review of literature was conducted in order to develop a philosophy and a set of guidelines which will be used to develop a trigonometry unit. This author combined the research based information from the review of literature with her own personal experience teaching high school geometry students, in order to develop the unit. In Chapter 4, a philosophy of teaching, based on the brain compatible paradigm, was presented. Lesson plans, graphic organizers, and activity worksheets were provided which incorporate the philosophy.

Peer Assessment

After completion of the geometry unit, this author asked several practicing educators (e.g., mathematics teachers and a mathematics department coordinator) to review the project and provide informal assessment. This author reflected on the feedback, and appropriate changes were incorporated into the final project. The specifics of the feedback received are discussed in Chapter 5.

Chapter Summary

An understanding of the human brain and how it functions in regard to the many facets of learning is important knowledge for educators to possess. Brain compatible teaching methods are methods to which all kinds of minds can be responsive and experience success. Therefore a trigonometry unit was developed which incorporates this important information. As described in Chapter 3, this author developed a trigonometry unit for the target audience based on the methods introduced. The unit was presented in detail in Chapter 4 and a discussion of the unit, after reflection on peer evaluation, follows in Chapter 5.

Chapter 4

RESULTS

The purpose of this project is to develop a trigonometry unit that is based on the findings from the current research on: (a) brain physiology, (b) adolescent learning, and (c) self-efficacy. With the use of research based methods, educators can help students develop a stronger foundation for the basic concepts of trigonometry. The use of hands-on projects, incorporated into the unit, will advance students' understanding and provide them with a sense of the real world applications of trigonometry.

A brain compatible learning environment includes activities in addition to the subject matter at hand. Early in the year students need to be informed as to how they learn, the natural learning process, and how the brain functions as an organ. An understanding of the function of the brain and the learning process allows students to realize that they are in control of their own learning. With this information students will have the ability to correctly evaluate their performance and adjust their behavior accordingly. Adolescents are yearning for control of their life and learning is one area where they can take full control. To address this issue, this author has included the following lesson, Lesson Plan #1 NHLP, which the teacher should present at the beginning of the first semester. Refer to Appendix E for handouts.

Lesson Plan #1 NHLP

Lesson Preparation

Unit Title	NHLP
Subject of Lesson	Natural Human Learning Process and the Human Brain
Duration	90 minutes
State Standards/ Benchmarks	Mathematics standards do not apply to this lesson.
Objective	Students will learn about how they learn and how the brain functions.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic
Materials Needed	Hand-outs: L.S.I. (Learning Scene Investigators) and Major Points About Learning (See Appendix E)

Lesson Delivery

Transition	
Follow-up	First lesson, no follow-up.
Essential Question Purpose/Motivation	How does one learn? Does everyone learn using a similar process? How does the brain work?
Activating Schema	Question #1 on the L.S.I. handout. Think of something that you are good at that you learned how to do outside of school such as a hobby, a sport, a household chore, an activity, repairing something, or anything else.
Instructional Plan	<u>65 min. - Part I</u>: Refer to the L.S.I. handout. The handout is intended as a note sheet for students to write down their thoughts. It does not need to be collected. 1. My Skill: Help student think of something they can do well if

they are having trouble (tying their shoes, riding a bike, driving a car, mowing the lawn, shooting a basketball, etc.. everyone knows how to do something well).

2. How I Learned it: Students should be able to come up with 4 or more steps.

3. & 4. Monitor groups.

5. Make a chronological list of the stages of learning.
On the blackboard write Stage 1, Stage 2, Stage 3, Stage 4, and so on. As students report the similarities, discuss where they fall in the list. The list will be similar to the following:

Stage 1: Motivation/Responding to stimulus in the environment: watched, observed, had to, interest, desire, curiosity.

Stage 2: Beginning Practice/Doing it: practice, practice, practice, trial and error, ask questions, consult others, basics, make mistakes, lessons, some success.

Stage 3: Advanced Practice/Increase of skill and confidence: practice, practice, practice, trial and error, some control, reading, encouragement, experiment, tried new ways, positive attitude, enjoyment, lessons, feedback, confidence, having some success, start sharing.

Stage 4: Skillfulness/Creativity: Practice, doing it one’s own way, feeling good about yourself, positive reinforcement, sharing knowledge, success, confidence.

Stage 5: Refinement/Further improvement: learning new methods, becoming second nature, continuing to develop, different from anyone else, creativity, independence, validation by others, ownership, habit, teaching.

Stage 6: Mastery/Application: greater challenges, teaching it, continuing improvement or dropping it, feeds into other interests, getting good and better and better, going to higher levels.

6. When the list is complete ask the students to raise their hand if they learned this way. If someone says they were good from the start, point out that they have a natural talent (aptitude) for the skill. Point out that all of the students are natural born learners and that the natural function of the brain is to learn. Students can be successful in this course just as they were outside of school because they will be provided with the same type of learning activities.

7. Ask each person to tell the class their skill so the class will see that they are all smart and everyone can do something well.

8. Discuss how the stages of learning relate to the following chart. Learning takes time and practice. The skill level will depend on how much time and practice is invested in learning the skill.

This activity was adapted from Smilkstein, 2003, p. 32-49

<u>20 min. - Part II:</u> The Brain
The teacher uses direct instruction for a brief lesson on the human brain. The following topics should be included.

The Brain Basics:
Hardest working organ in the body.
3% of the body's weight but uses 20% of its energy.
Needs water, oxygen and good nutrition to function well.

Brain Cells:
Composed of two types of cells 90% glia and 10% neurons.
Glia cells: transport nutrients, regulate immune system, form blood/brain barrier.
Neurons: information processing, conversion of electrical signals.

Neurons:
Receives signals from other cells.
The energy flows down the axon to the synapse.
The synapse is a space between cells with chemicals that transport the electrical impulses.
Many neurons are firing simultaneously.

Learning:
Learning requires groups of neurons.
When learning takes place the axon splits into branches called dendrites.
More connections make the brain more efficient.
Also, myelin forms around well used axons making it more efficient.
The myelination process increases by 100 percent in some parts of the adolescent brain.
It takes time for the body to make dendrites and form myelin.
When brain cells are not used they are lost due to pruning.

Chemicals in the Brain:
The brain is affected by chemicals produced by the body such as growth hormones which are produced in surges during adolescence.

	Testosterone in boys affects the fight-or-flight response, a surge of testosterone can cause anger or aggression. Estrogen and progesterone in girls can amplify emotion. Positive thinking produces pleasure chemicals in the brain. After the lecture the teacher should answer student questions and hand out the Major Points About Learning sheet.
Guided Practice	None
Check for Understanding	All students should be participating.
Closure	5 min. - Read the Major Points About Learning handout.
Differentiation	Shy students may need encouragement to participate. All student answers should be accepted and respected.
Independent Practice - Homework	Students take home the Major Points About Learning handout and put it somewhere in their room where they can read it daily.
Reflection on the lesson	

Classroom Environment to Promote Brain Compatible Learning

Prior to creating specific lesson plans that promote brain compatible learning, the teacher should consider a broad view of the characteristics of a brain compatible classroom. First, consider the fact that emotions are strongly intertwined with the learning process. The classroom should be a place where students feel respected and safe in taking risks. It is important to eliminate barriers to learning such as anxiety, stress, and fear. Positive thinking should be encouraged and conversely, negative thinking discouraged.

Additionally, there are other barriers to learning that may be found in a traditional classroom that should also be eliminated, such as memorization of large quantities of factual information, too many passive learning experiences, and inadequate involvement to facilitate the connection to new material. To avoid having students memorize large quantities of factual information, the teacher should determine the critical information that students need to memorize. For example, the teacher should determine which mathematical formulas are the most important for students to memorize and which formulas could be provided on a formula sheet when taking an exam. Lessons with too many passive learning experiences and inadequate student involvement should be replaced with lessons that follow the NHLP. First students explore the concept with individual work, (i.e., some type of activity to help them make an initial personal connection. Then they work in small groups where they are actively involved in both giving and receiving feedback. Whole group work should also be included for direct instruction, feedback, and closure.

The classroom environment should be enriched. An enriched environment includes activities that are challenging and novel, and involves students in interactive feedback. An enriched environment includes motor stimulation and a variety of approaches for problem solving such as demonstrations, models, discussion, and paper.

Lessons should reflect the strategies that strengthen student self-efficacy. Moderately challenging tasks can be assigned so students will experience success or mastery. Students should be grouped heterogeneously so lower self-efficacy students will be able to observe and get feedback from successful peer models. Notes and graphic organizers generated by the teacher should include a logical sequence of steps for students to follow when solving problems. The teacher should encourage students by telling them that they have the ability to learn the material successfully, provide guidance, and re-enforce the concept that continued effort and the correct use of strategies will lead to the students' success.

Finally, the five rules that the brain follows during the learning process should be incorporated in lesson plans; 1.) new learning is connected to something that is already known, 2.) people learn and remember by practice, 3.) learning requires interactive feedback with other individuals, 4.) when a skill is not used, the ability to perform that skill is diminished, and 5.) emotion and learning are inextricably bound together.

The Trigonometry Unit

The trigonometry unit that follows includes seven lesson plans. Each lesson is presented in two parts; 1.) lesson preparation, and 2.) lesson delivery. The graphic organizers for notes, practice problems, activities, handouts, and teacher notes that accompany the lessons are included in the Appendices. The trigonometry unit is generally taught at the end of the third quarter, therefore this author has assumed that students have mastered skills taught earlier in the year such as simplifying radicals and solving proportions.

Unit Plan for Trigonometry

Content Area Unit Title Grade Level	Mathematics - Geometry Trigonometry High School
Duration	Eight 90 minute periods
Mathematics State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. Describe geometric relationships algebraically. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Find and analyze relationships among geometric figures using transformations. Derive and use methods to measure area or regular and irregular geometric figures. Students use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known). 5 - Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems. Use appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision. Students determine the degree of accuracy of a measurement (for example, by understanding and using significant digits). 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Essential Question Purpose/Motivation	By the end of this unit students will understand the basic concepts of trigonometry and be able to use those concepts to solve real-world problems.

Prerequisite Knowledge	Students can solve proportions, simplify radical expressions, find the geometric mean, understand the properties of triangles, and understand similar triangles. Accommodations: the teacher gives individual students extra help during a free period and/or does a review activity or game with the class.
Unit Overview and Rationale	Students will learn about the natural learning process and how the brain functions in regard to learning. The unit will be taught using brain compatible teaching methods in order to increase student motivation and understanding. Students will understand the basic concepts of trigonometry and be able to use those concepts to solve real-world problems.
Teaching Strategy NHLP	Brain compatible teaching includes: 1.) individual work for making initial connections and practicing skills, 2.) small group work for active student participation, feedback, and practicing skills and, 3.) whole group work for direct instruction, feedback, and closure.
Materials Needed	3"x 5" blank index cards, rulers, scissors, protractors, pencils, red, blue & yellow markers, calculators, overhead, overhead pens, chalkboard, chalk, Geometry textbook, lined notebook paper, yard sticks or meter sticks, stapler, string, large paper clips, tape, straws, meter sticks and/or tape measures, clipboards, dry erase boards, markers and erasers.
Assessment/ Data Collection	The teacher will assess student understanding daily via homework checks, monitoring practice during class time, and exit tickets. Students will take a quiz in the middle of the unit. The unit exam will be the final assessment.
Reflection of the Unit	

Lesson Plan #1

Lesson Preparation	
Unit Title Textbook Section Subject of Lesson	Trigonometry 9.1 Similar Right Triangles
Duration	90 minutes
State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. Describe geometric relationships algebraically. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Find and analyze relationships among geometric figures using transformations. 5 - Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems. Use appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision. 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will learn the relationships between the right triangles formed when an altitude is drawn to the hypotenuse of a right triangle.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic

Materials Needed	Hand-outs: Activity #1 and Notes & Practice #1 (See Appendix F) Materials: 3"x 5" blank index cards, rulers, scissors, red, blue & yellow markers, calculators.

Lesson Delivery

Transition	<u>10 min. - Warm-up</u> (on overhead) - See teacher notes #1
Follow-up	First lesson, no follow-up.
Essential Question Purpose/Motivation	When an altitude is drawn to the hypotenuse of a right triangle, How many triangles are formed and how are the triangles related?
Activating Schema	<u>5 min. -</u> Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	<u>30 min. - Activity #1</u>: (See Appendix F) Model steps #1-7. Then students do #1-7 independently. Group students (3-4 per group) to complete #8-12. <u>15 min. -Notes & Practice (front)</u>: Read the theorems (whole group). Recap Theorem 9.1 which was explored in Activity #1. Theorem 9.2 & 9.3 are presented using examples #1-7 on the front. Model #1 then have students set-up #2. Show the correct set-up for #2 then have students solve the proportion. Show the correct solution. Repeat this process for #3 & 4 then for #5 & 6.
Guided Practice - Class work	<u>25 min. - Notes & Practice (back)</u> - Students complete problems #7-12 in small groups.
Check for Understanding Assessment/Evaluation	Teacher monitors working students and checks for correct answers. Give help and explanation as needed. <u>Notes & Practice (back)</u> - Students should check their answers with the key after completing the work. They should make corrections if needed.
Closure	<u>5 min. - Exit Ticket:</u> Ask students to: 1.) draw a right triangle then draw an altitude to the hypotenuse, and 2.) describe a relationship between the triangles. They should label the triangle as needed.

Differentiation	If needed: extra time, extra guidance with measuring and cutting for activity #1. Students who finish early may start the homework.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should reflect the lesson, range in difficulty, and include word problems.
Reflection on the lesson	

Lesson Plan #2

Lesson Preparation

Unit Title Textbook Section Subject of Lesson	Trigonometry 9.2 Pythagorean Theorem
Duration	90 minutes
State Standards/ Benchmarks	2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. Describe geometric relationships algebraically. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Derive and use methods to measure area or regular and irregular geometric figures. 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will learn the Pythagorean Theorem and how to use it in problem-solving situations.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic
Materials Needed	Hand-outs: Activity #2 and Notes & Practice #2 (See Appendix G) Materials: 3"x 5" blank index cards, rulers, scissors, lined notebook paper, yard sticks or meter sticks.

Lesson Delivery

Transition	<u>10 min. - Warm-up</u> (on overhead) - See teacher notes #2 Record homework and check for areas of misunderstanding.

Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	What is the relationship between the three sides of a right triangle? How can we use this relationship to find unknown distances and areas?
Activating Schema	5 min. - Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	15 min. - Activity #2: Model steps or entire activity as needed. Students should work independently. 15 min. -Notes & Practice (front): Read the Pythagorean Theorem. Recap activity #2 which proved the theorem. Stress the importance of identifying the hypotenuse and legs before writing the equation.
Guided Practice - Class work	20 min. - Notes & Practice (back) - Students complete the problems in small groups. In problem #12 they need to use unit conversion.
Check for Understanding Assessment/Evaluation	Teacher monitors working students and checks for correct answers. Give help and explanation as needed. Notes & Practice (back) - Students should check their answers with the key after completing the work. They should make corrections if needed.
Closure	15 min. - Exit Ticket: Students work in pairs. Ask students to find a rectangle somewhere in the room then 1.) measure the length and width, 2.) use the Pythagorean theorem to determine the length of the diagonal (they may need to convert units), 3.) measure the diagonal and compare it to the answer for #2.
Differentiation	If needed: a copy of the notes, extra time, extra guidance with cutting and tracing for activity #2. Students who finish early may start the homework.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should reflect the lesson, range in difficulty, and include word problems.

Reflection on the lesson	

Lesson Plan #3

Lesson Preparation

Unit Title Textbook Section Subject of Lesson	Trigonometry 9.4 Special Right Triangles
Duration	90 minutes
State Standards/ Benchmarks	2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. Describe geometric relationships algebraically. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Students use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will learn the relationships between the sides of 45˚- 45˚- 90˚ triangles and 30˚- 60˚- 90˚ triangles. They will learn how the relationships can be used with squares and equilateral triangles.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic
Materials Needed	Hand-outs: Notes & Practice #3 and Quiz (See Appendix H) Materials: stapler, dry erase boards, markers and erasers.

Lesson Delivery

Transition	<u>10 min. - Warm-up</u> (on overhead) - See teacher notes #3 Record homework and check for areas of misunderstanding.

Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	When only one side length is known on a 45˚- 45˚- 90˚triangle or a 30˚- 60˚- 90˚ what is a quick way to determine the other two sides?
Activating Schema	5 min. - Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	15 min. -Notes & Practice (front): Read Theorem 9.8. Show how the Pythagorean Theorem proves Theorem 9.8 (see notes on key). Note that the when a diagonal of a square is drawn two 45˚- 45˚- 90˚ triangles are formed. Read Theorem 9.9. Show how the Pythagorean Theorem proves Theorem 9.9 (see notes on key). Note that when an altitude is drawn in an equilateral triangle two 30˚- 60˚- 90˚ triangles are formed. Examples #1-8: Have students try to label the missing sides of #1 then show the correct answer. Do #2 side length needs to be rationalized. Have students try #3 then show the answer. Do #4. Have students try #5 then show the answer. Do #6. Have students try #7 and 8 then show the answer.
Guided Practice - Class work	20 min. - Notes & Practice (back) - Students complete problems #7-12 in small groups.
Check for Understanding Assessment/Evaluation	Teacher monitors working students and checks for correct answers. Give help and explanation as needed. Notes & Practice (back) - Students should check their answers with the key after completing the work. They should make corrections if needed. 10 min. White Board Review - Review special triangles and lesson #1-3 material. See teacher notes #3 for problems. Put one problem up at a time. Students do the problem on their dry erase board then hold it up for the teacher to check. 20 min. Partner Quiz - Students work with 1 or 2 others. Pair students heterogeneously. Explain to students that the partner quiz is a learning quiz. If a student does not understand how to

	solve a problem the other student(s) need to explain it. The students should discuss their answers and agree on the correct solution. Each student will complete a quiz. All quizzes in a group should have the same answers. When the group hands in their quizzes they will be stapled together (by the teacher, shuffle them). The quiz on the top will be graded and everyone in the group will receive the same grade.
Closure	The quiz serves as the closing activity.
Differentiation	If needed: extra time. Students who finish early may start the homework.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should reflect the lesson, range in difficulty, and include word problems.
Reflection on the lesson	

Lesson Plan #4

Lesson Preparation

Unit Title Textbook Section Subject of Lesson	Trigonometry 9.5 Trigonometric Ratios
Duration	90 minutes
State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Students use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known). 5 - Students determine the degree of accuracy of a measurement (for example, by understanding and using significant digits). 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will learn the trigonometric ratios; sine, cosine, and tangent. They will learn how to use the trigonometric ratios in problem-solving situations.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic

Materials Needed	Hand-outs: Notes #5, Practice #5, and Trigonometric Ratio Chart (See Appendix I) Materials: calculators.

Lesson Delivery

Transition	10 min. - Warm-up (on overhead) - See teacher notes #4 Record homework and check for areas of misunderstanding.
Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	How can one determine the height of an object, such as a building, using the angle of elevation (or depression) and the distance to the object.
Activating Schema	5 min. - Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	20 min. -Notes & Practice: Explain the material in the notes as shown on the key. Introduce SOH-CAH-TOA as a way to remember the trig ratios. Make up a story to go with the word such as: Has anyone ever heard of Pocahontas? Did you know she had a sister (SOH-CAH-TOA)? When labeling the vertices of first triangle in the notes (A, B, C), explain that the side across from the vertex A is labeled lower case a etc. When labeling the sides H (hypotenuse), O (opposite), and A (adjacent) stress that it is in relation to the designated acute angle. After explaining the special triangles, have students find the sin, cos, and tan of 45° and verify that it is the same decimal as was found in the example. Do this for 30° and 60° as well.
Guided Practice - Class work	25 min. - Notes & Practice - Students complete problems in small groups. Problem #1: have students do this individually before splitting into groups. Point out/or ask students if they notice the simplified ratios and decimals are the same as determined on the front of the notes. A 45° angle will always have the same ratio between its sides because all 45°- 45°- 90° triangles are similar.

	On the back of the practice, students should write the fraction for each answer, then fill in the letter at the bottom of the page. The back is self checking since they should get something that makes sense as the answers to the riddles.
Check for Understanding Assessment/Evaluation	Teacher monitors working students and checks for correct answers. Give help and explanation as needed. <u>Notes & Practice (back)</u> - Students should check their answers with the key after completing the work. They should make corrections if needed.
Closure	<u>5 min. - Exit Ticket:</u> 1.) Draw a right triangle and label one of the acute angles K then label the hypotenuse, opposite, and adjacent sides in reference to angle K. 2.) Write SOH-CAH-TOA and show the ratios that it represents.
Differentiation	If needed: extra time, copy of the notes. Students who finish early may start the homework.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should reflect the lesson, range in difficulty, and include word problems.
Reflection on the lesson	

Lesson Plan #5

Lesson Preparation

Unit Title Textbook Section Subject of Lesson	Trigonometry 9.6 Solving Right Triangles
Duration	90 minutes
State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Students use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known). 5 - Students determine the degree of accuracy of a measurement (for example, by understanding and using significant digits). 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will learn how to use the ratio of the sides of a right triangle to find the acute angles. They will also learn how to solve right triangles.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic

Materials Needed	Hand-outs: Activity #5, Notes #5, Practice #5 (See Appendix J), and Trigonometric Ratio chart. Materials: protractors, rulers, calculators.

Lesson Delivery

Transition	5 min. - Warm-up: Activity #5 Record homework and check for areas of misunderstanding
Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	How can ratio of the sides of a right triangle be used to find the acute angles? What is minimum information needed to find all of the sides and angles in a right triangle?
Activating Schema	5 min. - Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	10 min. -Activity #5: Students work individually. Model #1b pick a decimal ie. if the cos is .9613 find the angle. Have students look for .9613 under the cos column then look across to see what angle has that ratio (16˚). 10 min. -Notes & Practice (front): Example #2 remind students to round the decimal to 4 places. Instruct them to look on the Trigonometric Ratio Chart to find the decimal then look across to see which angle has that ratio. Most of the time the decimal will fall between two angles, in which case the angle can be approximated to the half degree. Using the trigonometric ratio chart for lesson #4 & #5 will help students avoid confusion over whether to use sin or $\sin^{-1}$ on the calculator. After students have had time to process these concepts, the calculator method can be introduced. 10 min. -Notes & Practice (back): When solving triangles students will need to draw from all of the skills learned in this unit. They will use SOH-CAH-TOA, Pythagorean Theorem, the sum of the angles in a triangle = 180˚ (learned previously), and inverse sin, cos, tan.

Guided Practice - Class work	25 min. - Notes & Practice (back) - Students work in small groups.
Check for Understanding Assessment/Evaluation	Teacher monitors working students and checks for correct answers. Give help and explanation as needed. Notes & Practice - Students should check their answers with the key after completing the work. They should make corrections if needed.
Closure	5 min. - Exit Ticket: Ask students to draw 2 right triangles then label each triangle with enough information to solve the triangle.
Differentiation	If needed: a copy of the notes, extra time, extra guidance with measuring and constructing the triangles in Activity #5. Students who finish early may start the homework.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should reflect the lesson, range in difficulty, and include word problems.
Reflection on the lesson	

Lesson Plan #6

Lesson Preparation	
Unit Title Textbook Section Subject of Lesson	Trigonometry Clinometer Activity
Duration	90 minutes
State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Students use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known). 5 - Students determine the degree of accuracy of a measurement (for example, by understanding and using significant digits). 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will apply their knowledge of trigonometry by constructing and using a clinometer to determine the height of a tree, or other tall object.
Teaching Strategy NHLP	Individual work, small groups, whole group
Modalities	Visual, auditory, kinesthetic

Materials Needed	Hand-outs: Activity #6 (See Appendix K) Materials: protractors, string, large paper clips, tape, straws scissors, meter sticks and/or tape measures, clipboards, calculators.

Lesson Delivery

Transition	5 min. - Warm-up (on overhead) - See teacher notes #6 Record homework and check for areas of misunderstanding.
Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	How can one use trigonometric ratios to determine the height of a tall tree using a tool to read the angle of elevation and a measuring tape?
Activating Schema	5 min. - Review warm-up questions. Ask student volunteers to write their answers on the board. Check for understanding.
Instructional Plan Refer to notes on key(s) and teacher notes.	30 min. - Activity #6: Construction of the Clinometer : Model the construction of the clinometer. Students work in pairs to construct a clinometer. When students are finished have them practice looking at things in the room, such as the top of the window, and reading the angle of elevation. Prior to the activity the teacher should choose an area outdoors with several tall objects such as a flag pole, trees, and light posts (see teacher notes). Draw a plan of the location of the objects on the blackboard and instruct students to choose one of the designated objects. Using the Clinometer: Take students outdoors to the pre-selected area help students locate the pre-selected items. Students complete the back of Activity #6. After returning indoors students should be instructed to write the calculated height of the object they chose on the board. Discuss the reasons for discrepancies in the heights, include accuracy in measurements and significant figures.

Guided Practice - Class work	35 min. - Students start reviewing for the unit test. Each student should look through their notes and write a problem that they think should be on the test. After the students have written a problem individually they get together in small groups. Each group should produce a question for each section of the unit (five questions) and a solution for each of the questions. Students may include any of the questions that they wrote individually.
Check for Understanding Assessment/Evaluation	Clinometer Activity: Teacher monitors working students and checks for correct set-up and calculations. Give help and explanation as needed. Review/Test Questions: Correct the questions produced by each group.
Closure	5 min. - Exit Ticket: Ask students to think what other ways the measurement of angles of elevation or depression could be used to find unknown distances.
Differentiation	If needed: extra time, extra guidance with construction of the clinometer and measuring.
Independent Practice - Homework	Textbook problems. Students should check the answers to the odd numbered problems in the back of the book as they finish each problem. The assigned problems should be review problems that will prepare students for the unit test.
Reflection on the lesson	

Lesson Plan #7

Lesson Preparation

Unit Title Textbook Section Subject of Lesson	Trigonometry Unit Assessment
Duration	50 minutes
State Standards/ Benchmarks	1 - Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems. 2 - Students use algebraic methods to explore, model, and describe patterns involving numbers and shapes in problem solving situations. 4 - Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems. Students use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known). 5 - Students determine the degree of accuracy of a measurement (for example, by understanding and using significant digits). 6- Students link concepts and procedures as they develop and use computational techniques in problem solving situations and communicate the reasoning used in solving these problems. Use ratios and proportions in problem-solving situations. Select appropriate algorithms for computing with real numbers in problem-solving situations.
Objective	Students will complete a summative assessment of the trigonometry unit.
Teaching Strategy NHLP	Individual work
Modalities	Visual, auditory, kinesthetic

Materials Needed	Hand-outs: Unit exam and Trigonometric Ratio Chart. Materials: calculators.

Lesson Delivery

Transition	5 min. - Record homework and check for areas of misunderstanding.
Follow-up	10 min. Display answers to even numbered problems for students to correct their work. Go over problems that students had trouble solving or didn't understand.
Essential Question Purpose/Motivation	Unit Exam
Activating Schema	5 min. - Students are instructed to review their notes.
Instructional Plan	35 min. - Unit exam. The exam should include both student generated and teacher generated problems and provide a range of difficulty.
Guided Practice - Class work	Skills review sheet. The teacher should prepare a review sheet which includes previously learned algebra skills. The problems should provide a range of difficulty.
Assessment/Evaluation	Correct the exam.
Differentiation	Extra time if needed. Students who finish early may work on the skills review sheet.
Independent Practice - Homework	Complete the skills review sheet.
Reflection on the lesson	

Additional Extensions

Additional extensions may also be included in the trigonometry unit. This author has previous experience as a land surveyor and included a demonstration of the equipment used in land surveying. An explanation of how trigonometry used in real life situations by land surveyors was followed by a hands-on experience for students. Students were instructed on the use of the equipment and each had a turn using the equipment to measure an angle. Most geometry teachers will not have surveying experience, but teachers can use their creativity to add an extension such as this to one of their geometry lessons. Teachers could ask for parent volunteers or call a local business to find someone who can demonstrate how geometry is used in the real world.

Chapter Summary

The purpose of this chapter was to present a trigonometry unit based on brain compatible learning. A broad view of the classroom environment which promotes brain compatible learning was included. The intent of the project was to present a unit on trigonometry that uses strategies that work with the natural learning process of the brain to increase student motivation to learn this concept and their understanding of the concept. A discussion concerning the goals achieved by this project, the limitations of the project, evaluation of the project, and recommendations will follow in Chapter 5.

Chapter 5

DISCUSSION

Introduction

The intent of this project was to equip the teachers of high school geometry with a unit on trigonometry that uses strategies that work with the natural brain process to promote successful learning. The trigonometry unit serves as a sample unit in which brain compatible teaching strategies have been applied. Teachers can transfer the strategies discussed in this unit to other units and subjects that they teach to increase their effectiveness. The completed project was reviewed by master teachers and their opinions and suggestion are described in the following chapter.

Goals Successfully Incorporated into the Project

A trigonometry unit was developed based on the findings from the current research on: (a) brain physiology, (b) adolescent learning, and (c) self-efficacy.
To promote successful learning the lesson plans incorporated the following five rules that the brain follows during the learning process; 1.) new learning is connected to something that is already known, 2.) people learn and remember by practice, 3.) learning requires interactive feedback with other individuals, 4.) when a skill is not used, the ability to perform that skill is diminished, and 5.) emotion and learning are inextricably bound together. The lesson plans also incorporated; 1.) strategies to strengthen self-efficacy, 2.) strategies to remove the barriers that impede the ability of the brain to learn

successfully, and 3.) the Colorado Model Content Standards for Mathematics for Grades 9-12.

Assessment and Strengths of the Research Project

Four experienced teachers reviewed this project, then met with the author to discuss the project's strengths and weaknesses and to make suggestions for improvements to the project. Teachers 1, 2, and 4 are high school mathematics educators. Teacher 3 is an elementary school teacher.

All four teachers had positive comments about the hands-on activities included in the lessons. They felt that the activities allowed students to make concrete connections to the lesson and provided a valuable kinesthetic experience. The teachers also felt that all of the lesson plans followed the NHLP and that the lessons were easy to follow. Teacher 1 commented that any teacher could pick them up, know the objective, and follow through.

Teachers 1 and 2 liked the idea of the exit ticket and made a few suggestions. Teacher 1 suggested the tickets should be graded or receive written comments, then be returned to the students. Teacher 2 suggested that the exit tickets could be more varied by: (1) making the exit ticket a writing assignment (i.e., have students explain their thinking in complete sentences), (2) have students write their answers on dry erase boards so the teacher could give immediate feedback, and (3) have students answer questions via clickers (in the clicker system questions are displayed using a computer and projector, the clickers send signals to the computer allowing students to get immediate feedback which can be displayed in several formats on the projector screen).

All four teachers also commented positively on Lesson #1 NHLP. Teacher 3, the elementary school teacher, plans to use the lesson with her 5th grade class. She suggested including a hand-out that students could take home and use to explain to their parents the function of the brain in relation to learning. She felt that the lesson on the brain and using brain compatible teaching methods complimented her current math program, Everyday Mathematics, and would help her students both in the current year and in the transition to middle school, which uses the Connected Mathematics program.

Teacher 2 suggested that the NHLP lesson should include visual aids (i.e., diagrams of the brain, and brain cells). Teacher 4 suggested that information about the brain could be included in lessons throughout the year and there could be teacher/student discussion concerning which stage of learning they are in. Also, teacher 4 felt that Lesson #1 NHLP addressed the Colorado Model Content Standards for Reading and Writing, which could be included in the lesson plan to address reading across the curriculum.

Two of the high school mathematics teachers suggested expanding two of the lessons. Teacher 1 suggested expanding Lesson #3, Special Right Triangles, to include an algebraic solution using the Pythagorean Theorem. To start, the legs of the 45˚- 45˚- 90˚ are each labeled x and the hypotenuse is labeled h. Using the Pythagorean Theorem $x^2 + x^2 = h^2$. Solving the equation for h results in $h = \sqrt{2}\,x$. For the 30˚- 60˚- 90˚, an altitude is constructed in an equilateral triangle, the short leg and hypotenuse of one of the 30˚- 60˚- 90˚ formed are labeled x and 2x respectively. The long leg (altitude) is labeled h. Using the Pythagorean Theorem, then solving for h, results in $h = \sqrt{3}\,x$.

Teacher 2 suggested adding more problems that require rationalizing to the practice assignment in Lesson #1, Similar Right Triangles.

Limitations of the Research Project

One of the limitations of the project is that this author assumed that students had mastered a certain amount of prerequisite material. The trigonometry unit is presented during the third quarter at the high school where this author teaches and was developed with this time frame in mind. The order in which concepts are presented in mathematics classes often varies from school to school so teachers may need to account for this before presenting the unit. Additionally, the lesson plans are tailored to the block schedule, 90 minute periods. The amount of time per class period also varies from one school to another, therefore, the lessons may need to be adjusted after taking this into consideration.

This project was intended to serve as a sample unit for teachers of other subjects and grade levels. High school teachers, particularly science teachers, may be able to transfer the information to the subjects that they teach, but middle school teachers would need to reflect on the particular needs of younger adolescent students in order to use this as a sample unit. Elementary school teachers may need to make more adjustments.

Also, throughout the school year, new students may join the class. These students will have missed the beginning of the year activity on the natural learning process and information on the brain. The teacher could review the NHLP information with the new student, but it may not have the same effect as participating in the activity with the class as a whole.

Recommendations for Additional Research

Although much of the information presented in the review of literature will be applicable to teachers at lower grade levels, these teachers would benefit from further investigation of the particular needs of the age group of the students that they teach. Also, teachers of different subjects would benefit from investigation into the examples that are provided in literature that pertain to the particular subject that they teach.

The review of literature touched on many of the concerns of adolescent learning, additional research that would benefit the reader include the following topics: (a) learning disabilities, (b) giftedness, (c) underachievement, and (d) teacher-student rapport.

Research Project Summary

A classroom teacher is accountable for instructing students who bring a wide variety of strengths and skills to their class. The benefit of using brain compatible teaching methods is in that they are suited for every type of learner. All students will benefit when a teacher aligns instruction with the learning process inherent in the human brain. This project provided one example of a brain compatible teaching unit. Teachers who are interested in providing their students with the natural learning opportunities that promote successful learning for all students will be wise to educate themselves about, and employ this paradigm.

REFERENCES

Abbott, J., & Ryan, T. (1999). Constructing knowledge, reconstructing schooling. *Educational Leadership, 57*(3), 66-69.

Accordino, D. B., Accordino, M. P., & Slaney, R .B. (2000). An investigation of perfectionism, mental health, achievement, and achievement motivation in adolescents. *Psychology in the Schools*, *37*(6), 535-544.

Anderson, A., Hattie, J., & Hamilton, R. J. (2005). Locus of control, self-efficacy, and motivation in different schools: Is moderation the key to success? *Educational Psychology*, *25*(5), 517-535. Retrieved March 28, 2007, from Academic Search Premier database.

Bandura, A. (1997). Self-efficacy. *Harvard Mental Health Letter*, *13*(9), 4-6. Retrieved March 28, 2007, from Academic Search Premier database.

Bandura, A., & Locke, E. A. (2003). Negative self-efficacy and goal effects revisited. *Journal of Applied Psychology*. *88*(1), 87-99.

Begley, S. (2007). *Train your mind, change your brain: How a new science reveals our extraordinary potential to transform ourselves*. New York: Ballantine Books.

Brandt, R. S. (1997). On using knowledge about our brain: A conversation with Robert Sylwester. *Educational Leadership, 54*(6), 16-19.

Caine, R. N., & Caine, G. (1991). *Making connections: Teaching and the human brain*. Alexandria, VA: Association for Supervision and Curriculum Development.

Caine, R. N., & Caine, G. (1997). *Education on the edge of possibility*. Alexandria, VA: Association for Supervision and Curriculum Development.

Caraway, K., Tucker, C. M., Reinke, W. M., & Hall, C. (2003). Self-efficacy, goal orientation, and fear of failure as predictors of school engagement in high schoolstudents. *Psychology in the Schools, 40*(4), 417-426.

Carroll, A., Durkin, K., Hattie, J., & Houghton, S. (1997). Goal setting among adolescents: A comparison of delinquent, at-risk, and not-at-risk youth. *Journal of Educational Psychology*, *89*(3), 441-450.

Colorado Department of Education (2005). *Colorado model content standards: Mathematics*. Retrieved March 28, 2007, from http://www.cde.state.co.us /cdeassess/documents/standards/math.pdf

Diagram of a neuron: Myelin sheath. Retrieved March 28, 2007, from http://www.usm .maine.edu/psy/broida/101/neuron.JPG

Diagram of the brain: Terms used to describe the parts of the brain. Retrieved March 28, 2007, from http://www.brainwaves.com/brain.html

D'Arcangelo, M. (1998). The brains behind the brain. *Educational Leadership, 56*(3), 20-25.

Ferrini-Mundy, J. et al. (2000). *Principals and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Introductory psychology image bank, neuron diagram. Retrieved March 28, 2007, from http://www.mhhe.com/socscience/intro/ibank/set1.htm

Jensen, E. (1998). *Teaching with the brain in mind.* Alexandria, VA: Association for Supervision and Curriculum Development.

Larson, R., Boswell, L., & Stiff, L. (2001a). *Geometry*. Evanston, IL: McDougal Littell.

Larson, R., Boswell, L., & Stiff, L. (2001b). *Geometry: Chapter 9 resource book.* Evanston, IL: McDougal Littell.

Levine, M. (2002). *A mind at a time.* New York: Simon & Schuster.

Marcy, S., & Marcy, J. (1996). *Algebra with pizzazz*. Chicago, IL: Creative Publications.

Margolis, H., & McCabe, P. P. (2006). Improving self-efficacy and motivation: What to do, what to say. *Intervention in School and Clinic, 41*(4), 218-227.

Marzano, R. J. et al. (1997). *Dimensions of thinking: A framework for curriculum and instruction*. Alexandria, VA: Association for Supervision and Curriculum Development.

Plumb bob. Retrieved July 9, 2007, from www.mainetechnicalsource.com /fieldsupplies.html

Pythagorean theorem: Proof #9. Retrieved July 9, 2007, from http://www.cut-the-knot.org/pythagoras/index.shtml#9

Smilkstein, R. (2003). *We're born to learn*. Thousand Oaks, CA: Corwin Press.

Sylwester, R. (1995). *A celebration of neurons: An educator's guide to the human brain*. Alexandria, VA: Association for Supervision and Curriculum Development.

Trigonometric Ratios. Retrieved July 9, 2007, from http://www.mels.gouv.qc.ca/publications/archives_dfgj/eval_2005/math416a_05.pdf

Trusty, J. (2000). High educational expectations and low achievement: Stability of educational goals across adolescence. *The Journal of Educational Leadership*, *93*(6), 356-365.

Urdan, T., & Mestas, M. (2006). The goals behind performance goals. *Journal of Educational Psychology*, *98*(2), 354-365.

Walsh, D. (2004). *Why do they act that way: A survival guide to the adolescent brain for you and your teen*. New York: Free Press.

White, W. F. (1996). What every teacher should know about the functions of learning in the human brain. *Education*, *117*(2), 290-296.

Wolfe, P., & Brandt, R.S. (1998). What do we know from brain research. *Educational Leadership*, *56*(3), 8-13.

Wolfe, P. (2001). *Brain matter: Translating research into classroom practice.* Alexandria, VA: Association for Supervision and Curriculum Development.

APPENDIX A

Three Types of Equipment Used in Brain Research

Three Types of Equipment Used in Brain Research

Type 1: Machines that measure chemical composition of cells and neurotransmitters	
CAT scan Computerized Axial Tomography	Multiple x-rays; Responds to density of tissue; Provides depth-of-field and clear cross-sectional views; Bones are dark, soft tissue is gray.
MRI Magnetic Resonance Imaging	Focus on soft tissue; Reverse image of CAT scan; Responds to chemical differences in soft tissue; Provides clear image of chemical composition of the brain; Can observe brain activity on t.v. monitor while subject is carrying out cognitive activity.
NMRI Nuclear Magnetic Resonance Imagery	30,000 times faster than MRI; Captures image every 50 milliseconds; Measures sequence of thinking across very narrow areas of the brain.
Spectrometers	Measures specifics of neurotransmitters as an activity occurs.
Type 2: Machines that measure electrical transmission of information along neuronal fibers and the magnetic fields that brain activity generates	
EEG Electro- encephalogram	In use for over 50 years; Shows patterns in electrical transmission of information in an active brain; Produces squiggly lines on moving paper which are not easily interpreted.
SQUID Superconducting Quantum Interference Device	More advanced technology than EEG; Uses small magnetic fields produced by the brain.
BEAM Brain Electrical Activity Mapping	Another major advance in technology; Records electrical activity from precisely defined areas; Uses color gradations to produce an easily interpreted graphic representation of the cerebral cortex of neurotransmitters as an activity occurs.
Type 3: Machine that measures distribution of blood through the brain as it replenishes energy used in electro-chemical activity	
PET Positron Emission Tomography	Uses radioactive materials to monitor unequally distributed patterns in blood flow in the brain; Traces sequential changes in brain energy use as various parts of the brain are activated shows how and where the brain processes a series of events. (Jensen, 1998; Sylwester, 1995)

APPENDIX B

The Neurodevelopmental Systems as They Translate into the Aims of Education

The Neurodevelopmental Systems as They Translate into the Aims of Education

Neural Developmental System	Aim of Education
Attention: Mental Energy Control	To educate students so they can reliably concentrate their mental resources and be capable of expending adequate work effort.
Attention: Intake Control	To educate students to think about what's important, to delay gratification, and to become active processors of information.
Attention: Output Control	To educate students to be reflective, to slow down and think through alternatives, to unite previous experience with foresight and vision.
Sequential Ordering	To educate students to be wise consumers of time, to understand how to think and act in a step-by-step fashion.
Spatial Ordering	To educate students to make good use of mental imagery and analogy, to engage in some productive and attractive nonverbal thinking.
Memory	To educate students to be thoughtful and systematic in managing their memory files and be able to merge understanding with remembering.
Language	To educate students to derive gratification and knowledge from language input and to become effective verbal communicators.
Motor	To educate students regarding the ways in which they can achieve a satisfying level of motor effectiveness.
Social Thinking	To educate students to understand and practice effective interpersonal skills and to be tolerant to differences in social values and styles.
Higher Thinking	To educate students as thinkers, so they become adept conceptualizers, creators, problem solvers, and critical analysts (Levine, 2002, pp. 316-317).

APPENDIX C

What One Would See within Schools for All Kinds of Minds

What One Would See Within Schools for All Kinds of Minds

- Teachers who are well versed in neurodevelopmental function and such serve as the lead local learning experts.
- Teachers who observe, describe, and respond to the neurodevelopmental observable phenomena of their students.
- Teachers who base their own teaching methods on their understanding of how learning works.
- Students who are learning about learning while they are learning.
- Students who gain insight into and are able to track their own evolving neurodevelopmental profiles.
- Students whose strengths have been properly identified and cultivated.
- Students who respect students whose neurodevelopmental profiles and personal backgrounds differ from their own.
- Parents who collaborate with schools and join forces to create and sustain schools for all kinds of minds.
- Schools that celebrate and foster neurodevelopmental diversity.
- Schools in which all students acquire and build unique expertise, maintain collections, and develop their affinities.
- Schools that make available multiple educational pathways.
- Schools that stress long-term projects over rapidly executed activities.
- Schools that help kids blaze their own trails for motor success, creativity, and community service.
- Schools that create and maintain an educational plan for each student.
- Schools that refuse to label their students.
- Schools where kids can learn and work at their own natural pace.
- Schools that offer a range of ways in which students can reveal their knowledge and academic accomplishments.
- Schools that seek to be far less judgmental of students.
- Schools that provide students with mentors from the faculty or from the community.
- Schools that help to educate parents about neurodevelopmental function and a mind at a time (Levine, 2002, pp. 334-335).

APPENDIX D

Colorado Model Content Standards for Mathematics, Grades 9-12

Standard 1
Students develop number sense and use numbers and number relationships in problem-solving situations and communicate the reasoning used in solving these problems.
GRADES 9-12
1. demonstrate meanings for real numbers, absolute value, and scientific notation using physical materials and technology in problem-solving situations;
2. develop, test, and explain conjectures about properties of number systems and sets of numbers; and
3. use number sense to estimate and justify the reasonableness of solutions to problems involving real numbers.

Standard 2
Students use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.
GRADES 9-12
1. model real-world phenomena (for example, distance versus-time relationships, compound interest, amortization tables, mortality rates) using functions, equations, inequalities, and matrices;
2. represent functional relationships using written explanations, tables, equations, and graphs, and describing the connections among these representations;
3. solve problems involving functional relationships using graphing calculators and/or computers as well as appropriate paper-and-pencil techniques;
4. analyze and explain the behaviors, transformations, and general properties of types of equations and functions (for example, linear, quadratic, exponential); and
5. interpret algebraic equations and inequalities geometrically and describing geometric relationships algebraically.

Standard 3
Students use data collection and analysis, statistics, and probability in problem-solving situations and communicate the reasoning used in solving these problems.
GRADES 9-12
1. design and conduct a statistical experiment to study a problem, and interpret and communicate the results using the appropriate technology (for example, graphing calculators, computer software);
2. analyze statistical claims for erroneous conclusions or distortions;
3. fit curves to scatter plots, using informal methods or appropriate technology, to determine the strength of the relationship between two data sets and to make predictions;
4. draw conclusions about distributions of data based on analysis of statistical summaries (for example, the combination of mean and standard deviation, and differences between the mean and median);

5. use experimental and theoretical probability to represent and solve problems involving uncertainty (for example, the chance of playing professional sports if a student is a successful high school athlete); and
6. solve real-world problems with informal use of combinations and permutations (for example, determining the number of possible meals at a restaurant featuring a given number of side dishes).

Standard 4
Students use geometric concepts, properties, and relationships in problem-solving situations and communicate the reasoning used in solving these problems.
GRADES 9-12
1. find and analyze relationships among geometric figures using transformations (for example, reflections, translations, rotations, dilations) in coordinate systems;
2. derive and use methods to measure perimeter, area, and volume of regular and irregular geometric figures;
3. make and test conjectures about geometric shapes and their properties, incorporating technology where appropriate; and
4. use trigonometric ratios in problem-solving situations (for example, finding the height of a building from a given point, if the distance to the building and the angle of elevation are known).

Standard 5
Students use a variety of tools and techniques to measure, apply the results in problem-solving situations, and communicate the reasoning used in solving these problems.
GRADES 9-12
1. measure quantities indirectly using techniques of algebra, geometry, or trigonometry;
2. select and use appropriate techniques and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements; and
3. determine the degree of accuracy of a measurement (for example, by understanding and using significant digits).
4. demonstrate the meanings of area under a curve and length of an arc.

Standard 6
Students link concepts and procedures as they develop and use computational techniques, including estimation, mental arithmetic, paper-and-pencil, calculators, and computers, in problem solving situations and communicate the reasoning used in solving these problems.
GRADES 9-12
1. use ratios, proportions, and percents in problem-solving situations;
2. select and use appropriate algorithms for computing with real numbers in problem-solving situations and determine whether the results are reasonable; and
3. describe the limitations of estimation, and assess the amount of error resulting from estimation within acceptable limits (Colorado Department of Education, 2005, pp. 5-10).

APPENDIX E

Learning Scene Investigators and Major Points About Learning Handouts

for NHLP Lesson

L.S.I. (Learning Scene Investigators)

1. My Skill: ___________________________________
Something that you are good at that you learned how to do outside of school such as a hobby, a sport, a household chore, an activity, repairing something, or anything else.

2. How I Learned it: Think back to the time before you learned your skill. Write the steps that you took to get from not being able to do it, to being able to do it well. Include how you felt during some of the stages.

3. Small Groups: (3-4 people per group). Introduce yourself then tell the others what your skill is and the steps you took to be able to do it well.

4. After everyone has had a chance to talk about their skill, discuss the similarities that you had in learning your skills.

Similarities:

5. Class discussion. We will discuss the results and list the stages of learning on the board.

(Adapted from Smilkstein, 2003, p. 32-39)

Major Points About Learning

1. Your brain was born to learn, loves to learn, and knows how to learn.

2. You learn what you practice.

> Practice is making mistakes, correcting mistakes, learning from them, and trying over, again and again.
>
> Making and learning from mistakes is a natural and necessary part of learning.

3. You learn what you practice because when you are practicing your brain is growing new dendrites and connecting them at synapses. This *is* what learning is.

4. Learning takes time because you need time to grow and connect dendrites.

5. If you don't use it, you can lose it. Dendrites and synapses can begin to disappear if you don't use them (if you don't practice what you have learned).

6. Your emotions affect your brain's ability to learn, think, and remember.

> Self-doubt, fear, etc., prevent your brain from learning, thinking, and remembering.
>
> Confidence, interest, etc., help your brain learn, think, and remember.

7. Remember, you are a natural born learner.

(Smilkstein, 2003, p. 103)

APPENDIX F

Similar Right Triangles

Lesson Plan #1: Activity #1 Investigating Similar Right Triangles

Materials Needed: 3"x 5" blank index card, ruler, scissors, pencil, 3 markers (red, blue & yellow), calculator.

1. Draw a diagonal from one corner of the index card to the opposite corner. Be as accurate as possible.

2. Cut the card along the diagonal line. You now have 2 congruent triangles.

3. On one of the triangles draw an altitude from the right angle to the hypotenuse. Use the other triangle to help you draw the 90° angle

4. Cut the triangle along the altitude. You now have 3 right triangles.

5. Measure the sides of each triangle (use cm. to the nearest tenth).

6. Trace each of the triangles separately in the space below. Write the dimensions found in #5 on each triangle below. Then label the sides of the triangles: short leg, long leg, and hypotenuse.

Key Lesson Plan #1: Activity #1 Investigating Similar Right Triangles

Materials Needed: 3"x 5" blank index card, ruler, scissors, pencil, 3 markers (red, blue & yellow), calculator.

1. Draw a <u>diagonal</u> from one corner of the index card to the opposite corner. Be as accurate as possible.

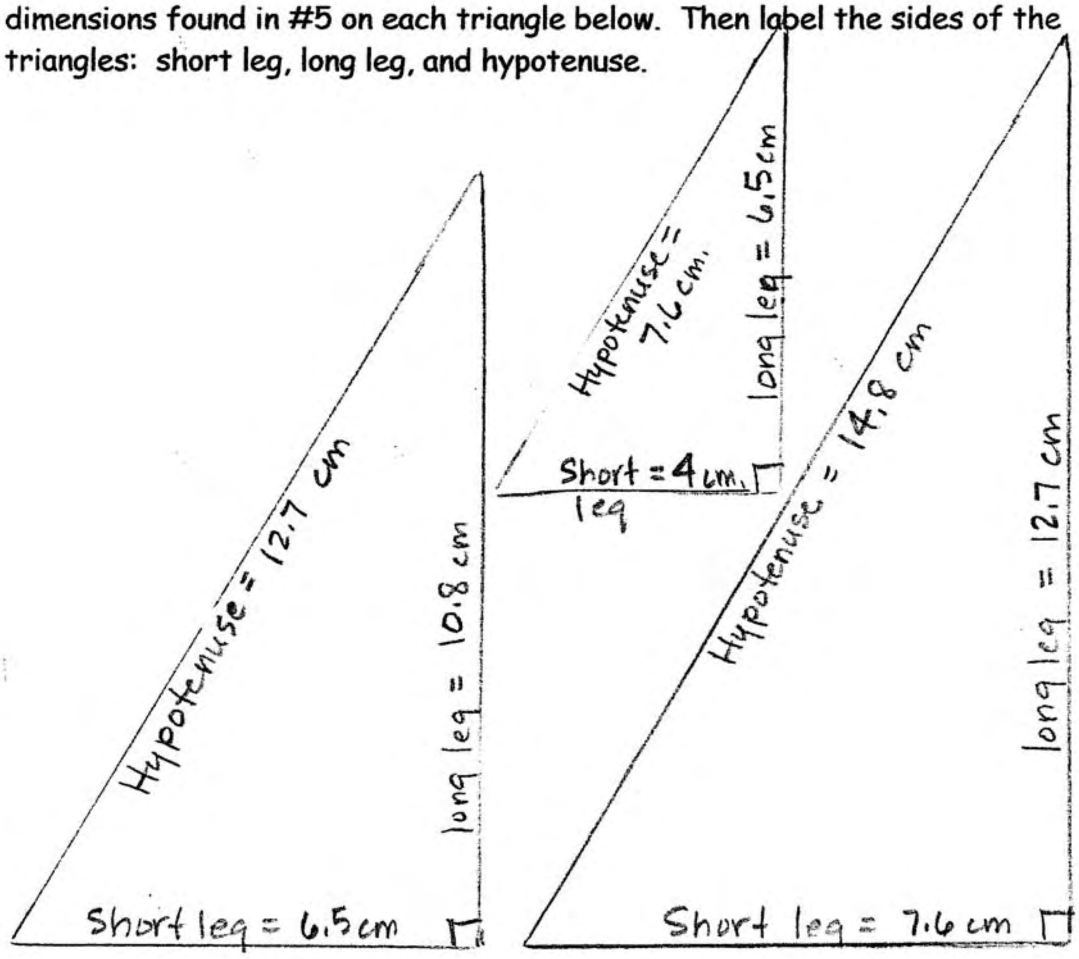
2. Cut the card along the diagonal line. You now have 2 congruent triangles.

3. On one of the triangles draw an <u>altitude</u> from the right angle to the <u>hypotenuse</u>. Use the other triangle to help you draw the 90° angle

4. Cut the triangle along the altitude. You now have 3 right triangles.

5. Measure the sides of each triangle (use cm. to the nearest tenth).

6. Trace each of the triangles separately in the space below. Write the dimensions found in #5 on each triangle below. Then label the sides of the triangles: short leg, long leg, and hypotenuse.



7. Use colored markers. Color the hypotenuse of each triangle red, color the long leg blue and color the short leg yellow.

8. With the largest triangle on the bottom investigate the different ways that the triangles can fit or stack inside one another. Draw a sketch of each.

9. Are the corresponding angles congruent?

10. How do you know that the triangles similar?

11. Fill in the table below to show the sides of the triangles are proportional. Use your measurements from #5.

Write the ratio as a fraction and a decimal

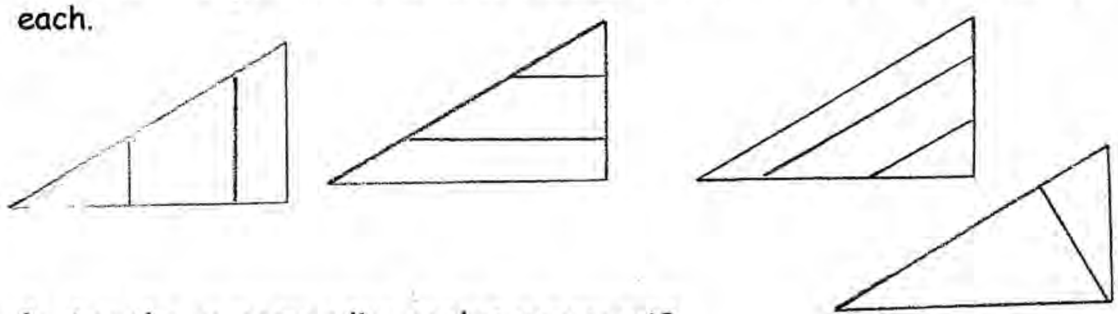
proportion	small triangle	medium triangle	large triangle	Are the proportions the same?
short leg / long leg	______ =	______ =	______ =	
short leg / hypotenuse	______ =	______ =	______ =	
long leg / hypotenuse	______ =	______ =	______ =	

12. What other proportions could you have used ?

Key

7. Use colored markers. Color the hypotenuse of each triangle red, color the long leg blue and color the short leg yellow.

8. With the largest triangle on the bottom investigate the different ways that the triangles can fit or stack inside one another. Draw a sketch of each.



9. Are the corresponding angles congruent?

Yes

10. How do you know that the triangles similar?

The corresponding angles are congruent

11. Fill in the table below to show the sides of the triangles are proportiona Use your measurements from #5.

Write the ratio as a fraction and a decimal

proportion	small triangle	medium triangle	large triangle	Are the proportions the same?
$\frac{\text{short leg}}{\text{long leg}}$	$\frac{4}{6.5} = .6$	$\frac{6.5}{10.8} = .6$	$\frac{7.6}{12.7} = .6$	Yes .6
$\frac{\text{short leg}}{\text{hypotenuse}}$	$\frac{4}{7.6} = .5$	$\frac{6.5}{12.7} = .5$	$\frac{7.6}{14.8} = .5$	Yes .5
$\frac{\text{long leg}}{\text{hypotenuse}}$	$\frac{6.5}{7.6} = .9$	$\frac{10.8}{12.7} = .9$	$\frac{12.7}{14.8} = .9$	Yes .9

12. What other proportions could you have used ?

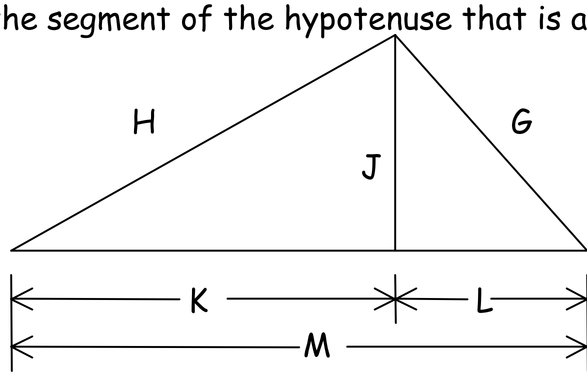
$\frac{\text{Long leg}}{\text{Short leg}}$ $\frac{\text{hypotenuse}}{\text{short Leg}}$ $\frac{\text{hypotenuse}}{\text{long leg}}$

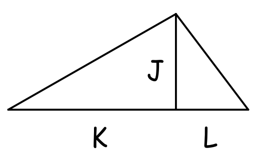
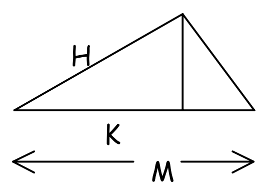
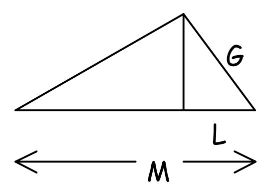
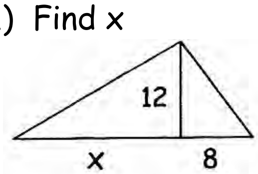
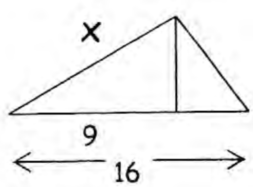
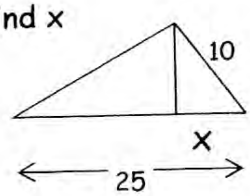
Lesson Plan #1: Similar Right Triangles Notes & Practice

Theorem 9.1 If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Theorem 9.2 In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments.

Theorem 9.3 In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



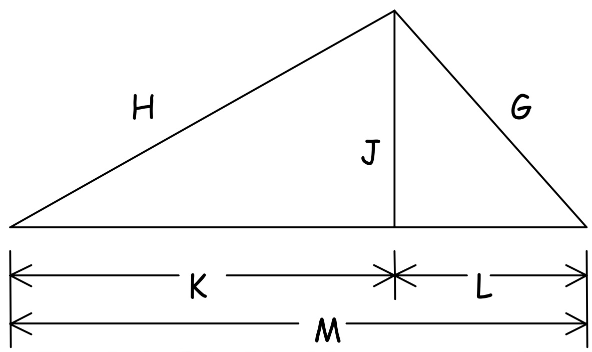
1.) Medium & Small 	3.) Medium & Large 	5.) Small & Large 
2.) Find x 	4.) Find x 	6.) Find x 

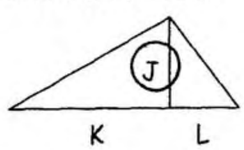
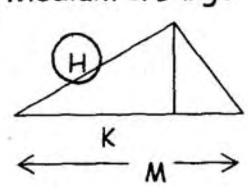
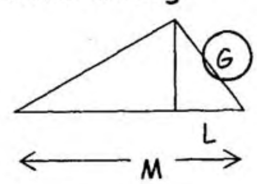
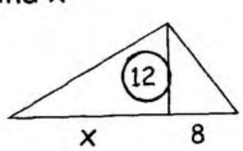
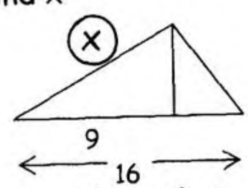
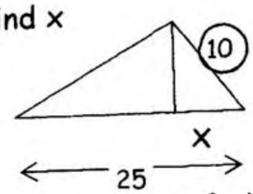
Key Lesson Plan #1: Similar Right Triangles   Notes & Practice

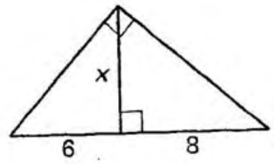
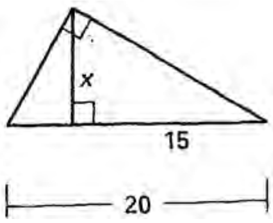
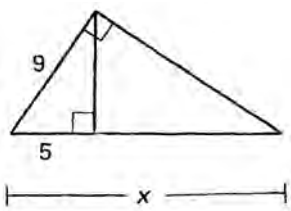
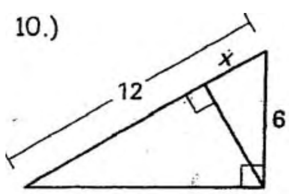
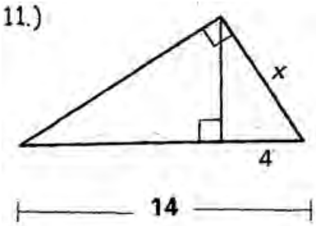
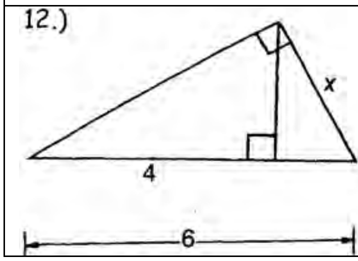
Theorem 9.1 If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Theorem 9.2 In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the *geometric mean* of the lengths of the two segments.

Theorem 9.3 In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the *geometric mean* of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

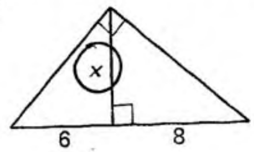
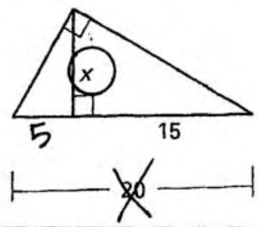
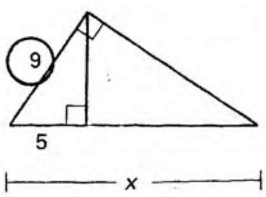
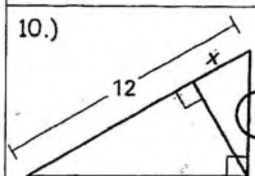
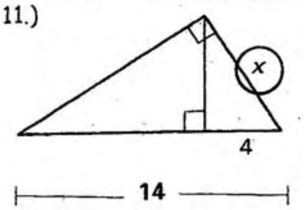
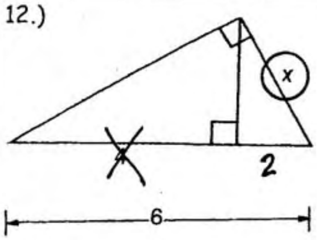


1.) Medium & Small  $\frac{K}{J}=\frac{J}{L}$	3.) Medium & Large  $\frac{K}{H}=\frac{H}{M}$	5.) Small & Large  $\frac{L}{G}=\frac{G}{M}$
2.) Find x  $\frac{x}{12}=\frac{12}{8}$ $8x=12(12)$ $8x=144$ $x=18$	4.) Find x  $\frac{9}{x}=\frac{x}{16}$ $x^2=9(16)$ $x^2=144$ $x=12$	6.) Find x  $\frac{x}{10}=\frac{10}{25}$ $25x=10(10)$ $25x=100$ $x=4$

Problem (circle common side)	Which 2 triangles?	Set-up and Solution
7.) 		
8.) 		
9.) 		
10.) 		
11.) 		
12.) 		

(Adapted from Larson, 2001b, p. 15)

Key

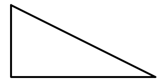
Problem (circle common side)	Which 2 triangles?	Set-up and Solution
7.) 	M & S	$\frac{6}{x} = \frac{x}{8}$ $x^2 = 48$ $x = 6.93$ OR $x = \sqrt{16}\sqrt{3}$ $x = 4\sqrt{3}$
8.) 	M & S	$\frac{5}{x} = \frac{x}{15}$ $x^2 = 75$ $x = 8.66$ OR $x = \sqrt{25}\sqrt{3}$ $x = 5\sqrt{3}$
9.) 	S & L	$\frac{5}{9} = \frac{9}{x}$ $5x = 81$ $x = 16.2$
10.) 	S & L	$\frac{x}{6} = \frac{6}{12}$ $12x = 36$ $x = 3$
11.) 	S & L	$\frac{4}{x} = \frac{x}{14}$ $x^2 = 56$ $x = 7.48$ OR $x = \sqrt{4}\sqrt{14}$ $x = 2\sqrt{14}$
12.) 	S & L	$\frac{6}{x} = \frac{x}{2}$ $x^2 = 12$ $x = 3.46$ OR $x = \sqrt{4}\sqrt{3}$ $x = 2\sqrt{3}$

Lesson Plan #1: Teacher Notes

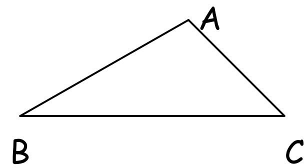
Investigating Similar Right Triangles

Warm-up

1.) label the legs and the hypotenuse of the right triangle.



2.) Draw an altitude from point A to side BC

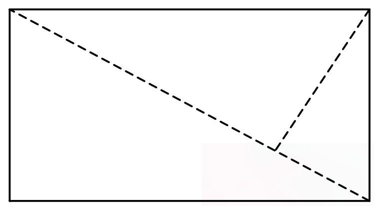


3.) If 2 triangles are similar their angles are ___ and their sides are ___ .

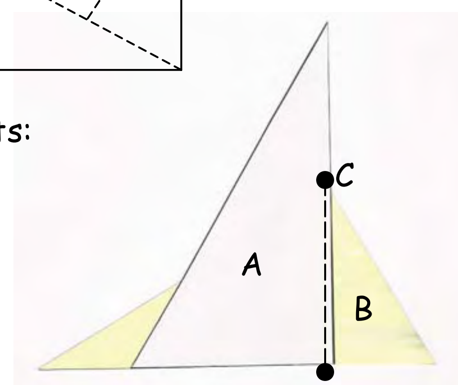
4.) Find the geometric mean of 3 and 12.

Activity #1

The 3"x5" card should be cut as shown in the diagram.



Step 3.) Model this procedure for students:
Line up the bottom of the triangles.
Move triangle A so that it lines up with the vertex of triangle B at point C.
Then draw the altitude.



Notes & Practice

Before the notes have the students set the triangles on their desk as shown in figure #2. The triangles can be referred to during the notes. Discuss how problems #1-6 in the Notes & Practice relate to the ratios found in Activity #1 step 11.

Figure #1 3 Similar Triangles

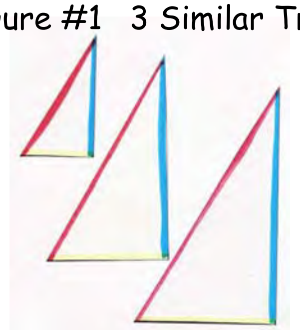
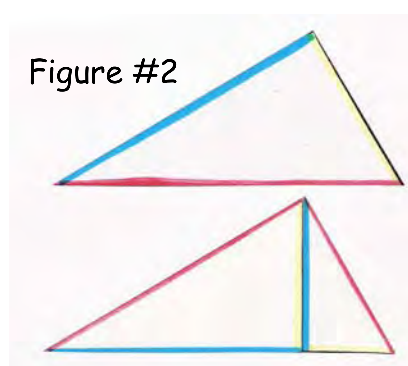


Figure #2

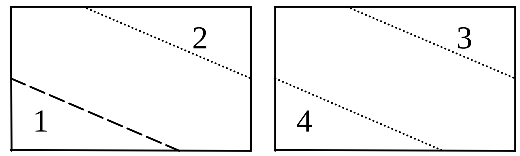


APPENDIX G

The Pythagorean Theorem

Lesson Plan #2: Activity #2 Investigating the Pythagorean Theorem

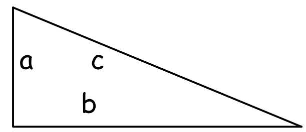
Materials needed: (2) 3"x 5" index cards, scissors, lined notebook paper.



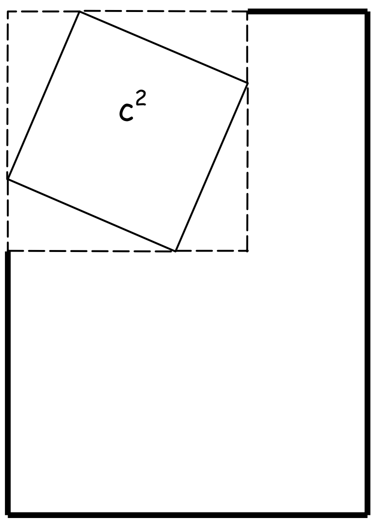
1. Cut a right triangle from the corner of one of the cards.

2. Trace the triangle and cut 3 more right triangles that are exactly the same size.

3. Label the sides of each triangle as follows. Label the: hypotenuse c, the long leg b, and the short leg a.



4. Arrange the triangles on a lined piece of paper as shown in the diagram (below left). Then trace the triangles (use the lines to keep the figure straight).



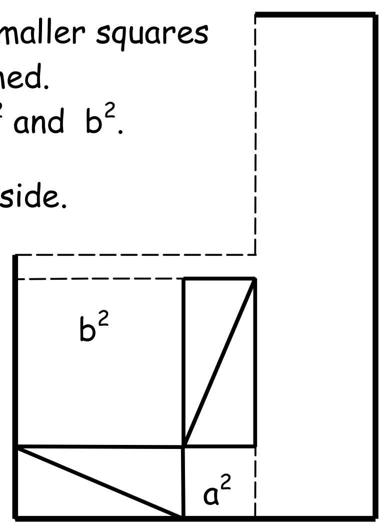
5. Remove the triangles. Label the square formed c^2 .

6. Cut out the large square formed by the triangles (shown by the dashed line).

7. Trace the large square on the notebook paper. On the new square arrange the triangles as shown below and trace the 2 smaller squares that are formed. Label them a^2 and b^2.

8. Place the two congruent large squares side by side. Notice that when the 4 original triangles are removed the result is:

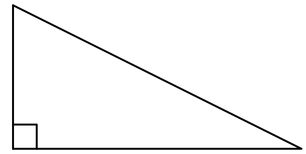
$$a^2 + b^2 = c^2$$



You have just proved the Pythagorean Theorem.
(Adapted from Pythagorean Theorem: Proof #9)

Lesson Plan #2: Pythagorean Theorem Notes

Theorem 9.4 Pythagorean Theorem
In a <u>right</u> triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



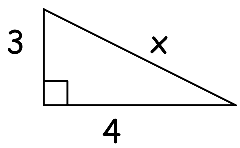
Step #1:

Step #2:

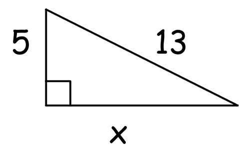
Step #3:

Pythagorean Triple:

1.) Finding the length of the hypotenuse.

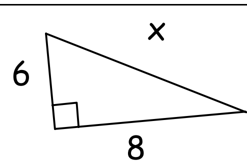


2.) Finding the length of a leg.

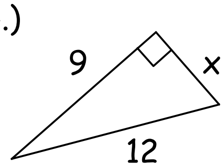


Find the value of x

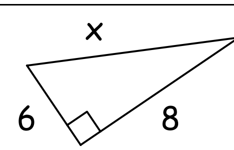
3.)



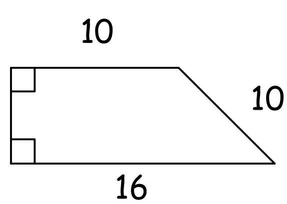
4.)



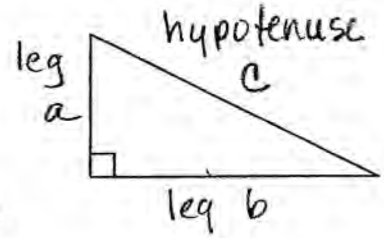
5.)



6.) Find the area of the figure.



Theorem 9.4 **Pythagorean Theorem**
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



Step #1: identify the hypotenuse & legs

Step #2: Substitute values into $a^2+b^2=c^2$

$$a^2+b^2=c^2$$

Step #3: Solve the equation.

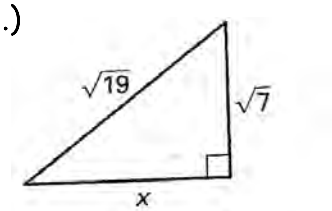
$$leg^2+leg^2=hyp.^2$$

Pythagorean Triple: Set of 3 positive integers that satisfy $a^2+b^2=c^2$ examples: 3,4,5 5,12,13 8,15,17

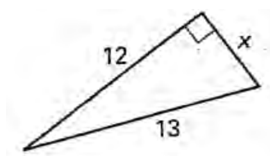
Lesson Plan #2 Practice

Find the unknown side length. Simplify answers that are radicals.
Tell whether the sides form a Pythagorean triple.

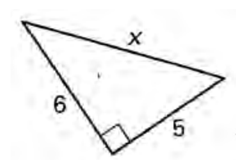
1.)



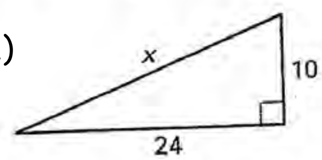
2.)



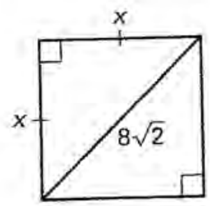
3.)



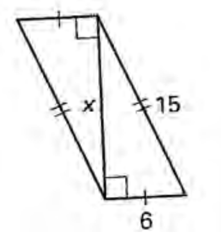
4.)



5.)

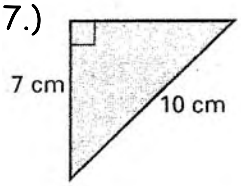


6.)

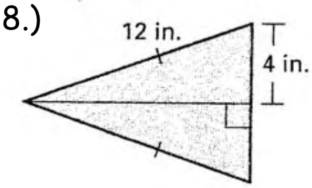


Find the area of the figure. Round decimal answers to the nearest tenth.

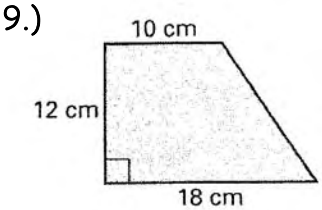
7.)



8.)



9.)



10. A 48 inch wide screen television means the measure along the diagonal is 48 inches. If the screen is square, what are the dimensions of the length and width?

11.) Each base on a standard baseball diamond lies 90 feet from the next. Find the distance the catcher must throw a baseball from 3 feet behind home plate to second base.

12.) A standard doorway measures 6 ft. 8 in. by 3 ft. What is the largest dimension that will fit through the doorway without bending?

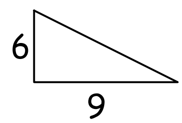
(Adapted from Larson, 2001b, p. 30)

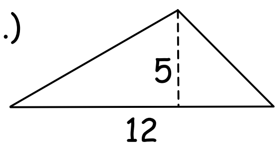
Lesson Plan #2: Pythagorean Theorem

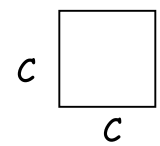
Teacher Notes

Warm-up

Find the area

1.) 

2.) 

3.) square 

Simplify the radicals

4.) $\sqrt{48}$ 5.) $5\sqrt{32}$ 6.) $\sqrt{500}$ 7.) $7\sqrt{63}$

Solve

8.) $\left(\sqrt{8}\right)^2 + x = \left(\sqrt{32}\right)^2$

Convert to inches or feet

9.) 14 ft 8 in. = ____ feet 10.) 18 ft 7 in = _____inches

Activity #2

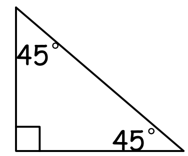
Step #4: Point out that the dimensions of the square is c x c therefore the area is c^2.

Step #7: Similar to step #4, b x b = b^2 and a x a = a^2.

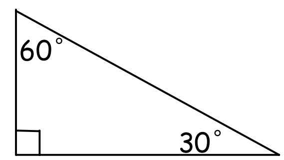
APPENDIX H

Special Right Triangles

Theorem 9.8 The 45°- 45°- 90° Triangle Theorem
In a 45°- 45°- 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

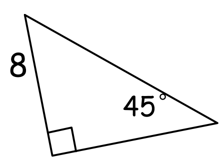


Theorem 9.9 The 30°- 60°- 90° Triangle Theorem
In a 30°- 60°- 90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

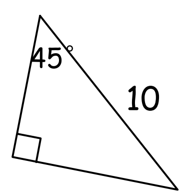


Label the triangles with the lengths of the missing sides.

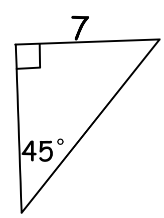
1.)



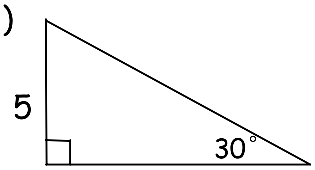
2.)



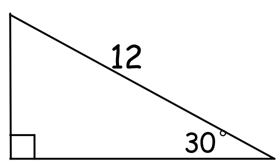
3.)



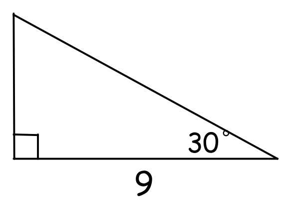
4.)



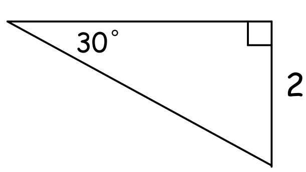
5.)



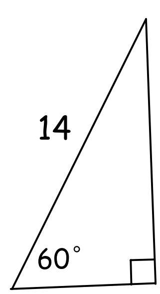
6.)



7.)

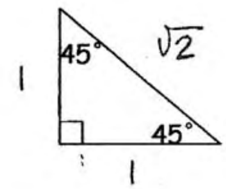


8.)



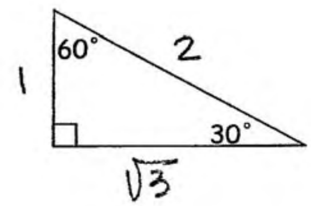
Key Lesson Plan #3: Special Right Triangles Notes

Theorem 9.8 The 45° - 45° - 90° Triangle Theorem
In a 45° - 45° - 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



$$1^2 + 1^2 = (\sqrt{2})^2$$
$$1 + 1 = 2 \checkmark$$

Theorem 9.9 The 30° - 60° - 90° Triangle Theorem
In a 30° - 60° - 90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

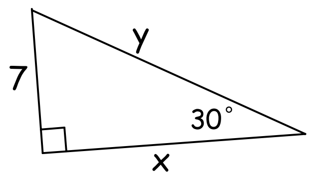


$$1^2 + (\sqrt{3})^2 = 2^2$$
$$1 + 3 = 4 \checkmark$$

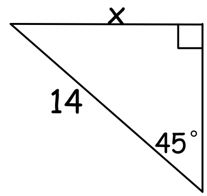
Lesson #3 Practice

Find the value of each variable. Write answers in simplest radical form.

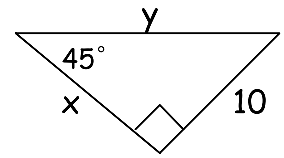
1.)



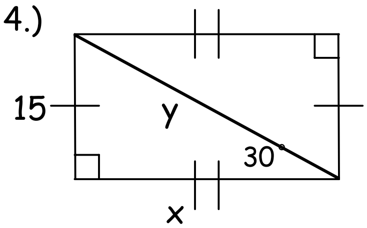
2.)



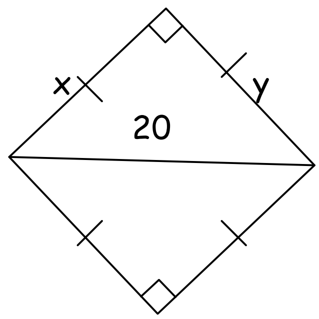
3.)



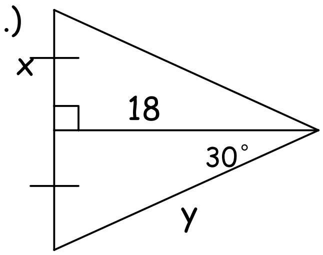
4.)



5.)



6.)



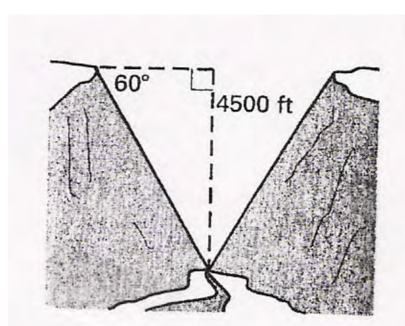
Sketch the figure that is described. Find the requested length.
Round decimals to the nearest tenth.

7.) The perimeter of a square is 20 cm.
Find the length of a diagonal.

8.) The side of an equilateral triangle is 36 cm.
Find the length of an altitude of the triangle.

9.) The diagonal of a square is 12 in.
Find the length of a side.

10.) The point on the edge of a symmetrical canyon is 4500 feet above a river that cuts through the canyon floor. The angle of depression from each side of the canyon to the canyon floor is 60°.



A.) Find the distance across the canyon.

B.) Find the length of the canyon wall
from the edge to the river.

C.) Is it more than a mile across the canyon?
(5280 feet = 1 mile)

(Adapted from Larson, 2001b, p. 58)

Lesson Plan #3: Special Triangles Teacher Notes

Warm-up

1.) Draw a square then draw a diagonal of the square. Label all of the angles formed in the figure that you drew.

2.) Draw an equilateral triangle then draw an altitude of the triangle. Label all of the angles formed in the figure that you drew.

3.) Rationalize the following.

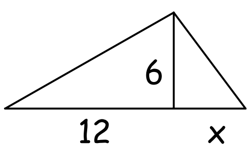
$\frac{15}{\sqrt{5}}$ $\frac{12}{\sqrt{3}}$ $\frac{4}{\sqrt{6}}$ $\frac{5}{\sqrt{7}}$

White Board Review

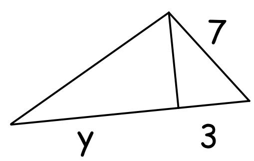
1.) Find the geometric mean of 8 and 24.

2.) Find the value of x.

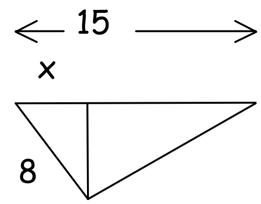
a.)



b.)



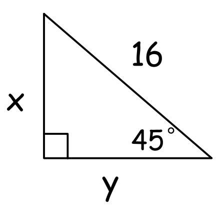
c.)



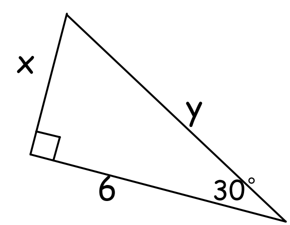
3.) Simplify the radical $\sqrt{32}$

4.) Use special triangles to find the value of the variables. Leave your answer in simplified radical form. Rationalize if needed.

a.)

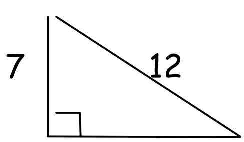


b.)

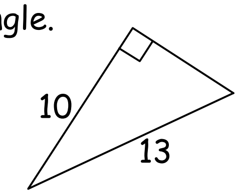


5.) Find the missing side of the right triangle.

a.)



b.)

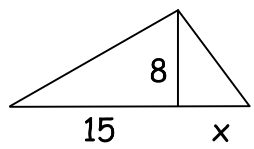


Lesson Plan #3: Partner Quiz on lesson #1-3

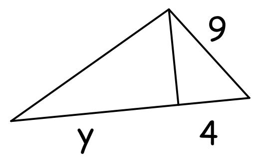
1.) Find the geometric mean of 9 and 36.

2.) Find the value of x.

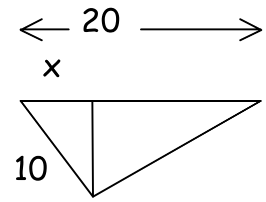
a.)



b.)



c.)

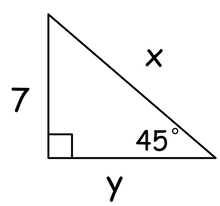


3.) Simplify the radical

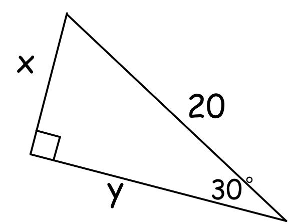
a.) $\sqrt{75}$ b.) $\sqrt{18}$

4.) Use special triangles to find the value of the variables. Leave your answer in simplified radical form. Rationalize if needed.

a.)



b.)

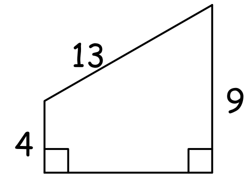


5.) The perimeter of a square is 24 cm.
Find the length of the diagonal.

6.) The length of an altitude of an equilateral triangle is 6 in.
Find the length of the sides of the triangle.

7.) A 16 ft. ladder is leaning against a wall. The base of the ladder is 4 ft. from the wall. How far up the wall does the ladder reach?

8.) Find the area of the figure shown.



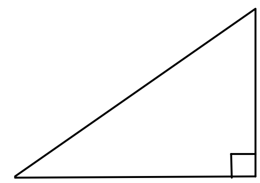
APPENDIX I

Trigonometric Ratios

Lesson Plan #4: Trigonometric Ratios Notes

Trigonometry:	Trigonometric ratios:

Sine sin A = ____________ = ___ =



Cosine cos A = ____________ = ___ =

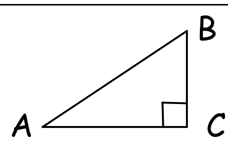
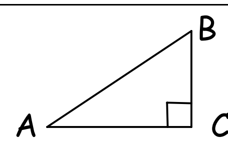
Tangent tan A = ____________ = ___ =

Step #1:

Step #2:

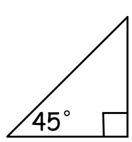
Step #3:

Examples

1.)	2.)
	
sin A =	sin B =
cos A =	cos B =
tan A =	tan B =

Special Triangles:

45°- 45°- 90° sin 45°= cos 45°= tan 45°=



Trigonometry:	Trigonometric ratios:
Measurement of triangles (Greek)	ratio of the lengths of 2 sides of a right triangle

Sine
SOH

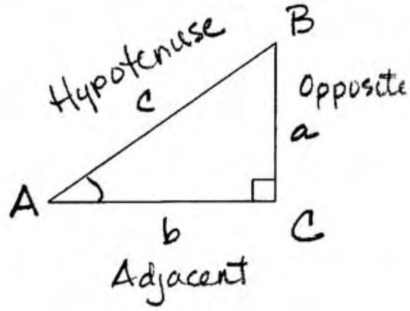
sin A = $\frac{\text{Opposite}}{\text{hypotenuse}} = \frac{O}{H} = \frac{a}{c}$

Cosine
CAH

cos A = $\frac{\text{Adjacent}}{\text{hypotenuse}} = \frac{A}{H} = \frac{b}{c}$

Tangent
TOA

tan A = $\frac{\text{Opposite}}{\text{Adjacent}} = \frac{O}{A} = \frac{a}{b}$



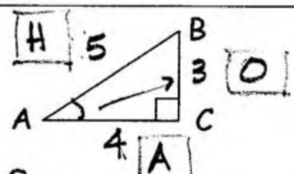
Step #1: Label the sides H, O, A

Step #2: Determine which sides using SOH-CAH-TOA

Step #3: Write the ratio as a fraction.
Convert to a decimal if needed.
(round to 4 places)

Examples

1.)

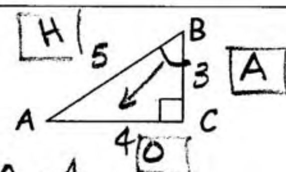


sin A = $\frac{O}{H} = \frac{3}{5} = .6$

cos A = $\frac{A}{H} = \frac{4}{5} = .8$

tan A = $\frac{O}{A} = \frac{3}{4} = .75$

2.)



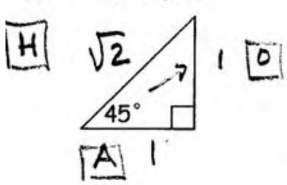
sin B = $\frac{O}{H} = \frac{4}{5} = .8$

cos B = $\frac{A}{H} = \frac{3}{5} = .6$

tan B = $\frac{O}{A} = \frac{4}{3} = 1.3333$

Special Triangles:

45° - 45° - 90°



sin 45° = $\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

Sin 45° ≈ .7071

cos 45° = $\frac{1}{\sqrt{2}}$

$\boxed{\frac{\sqrt{2}}{2}}$

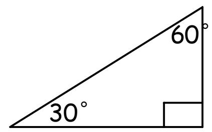
Cos 45° ≈ .7071

tan 45° = $\frac{1}{1}$

$\boxed{1}$

tan 45° = 1

30°- 60°- 90°



sin 30°= cos 30°= tan 30°=

sin 60°= cos 60°= tan 60°=

Word Problems - Using SOH - CAH - TOA

Step #1:

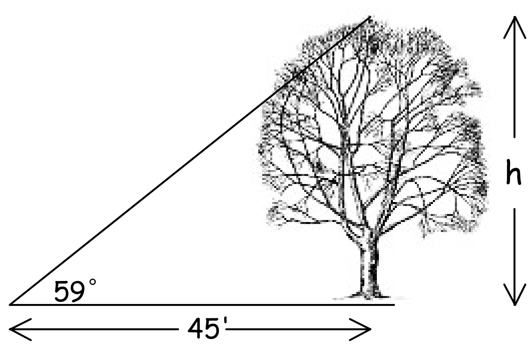
Step #2:

Step #3:

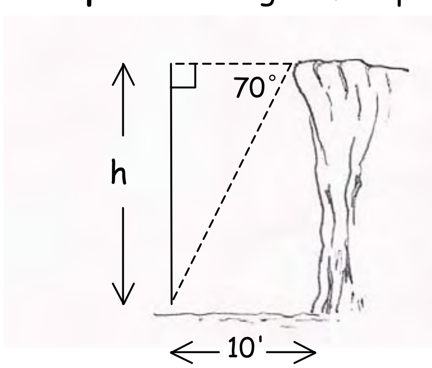
Step #4:

Step #5:

Example 1 Angle of Elevation: Find the height of the tree.

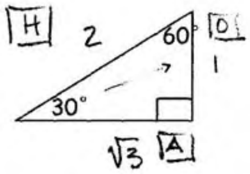


Example 2 Angle of Depression: Find the height of the cliff.



Key

30° - 60° - 90°

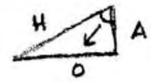


$\sin 30° = \frac{1}{2}$ $\quad$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 30° = .5$ $\quad$ $\cos 30° \approx .8660$ $\quad$ $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan 30° = .5774$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60° = \frac{1}{2}$ $\quad$ $\tan 60° = \frac{\sqrt{3}}{1}$



$\sin 60° \approx .8660$ $\quad$ $\cos 60° = .5$ $\quad$ $\tan 60° \approx 1.7321$

Word Problems - Using SOH - CAH - TOA

Step #1: Use the given information to draw & label a sketch

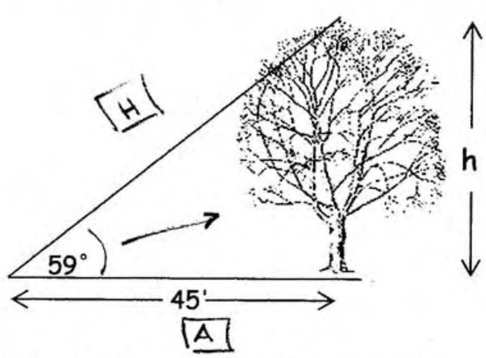
Step #2: Label the sides H, O, A

Step #3: Determine the trig. function Using SOH-CAH-TOA

Step #4: write the ratio using trig. function & info given.

Step #5: Solve for the unknown value

Example 1 Angle of Elevation: Find the height of the tree.



TOA $\quad \tan = \frac{O}{A}$

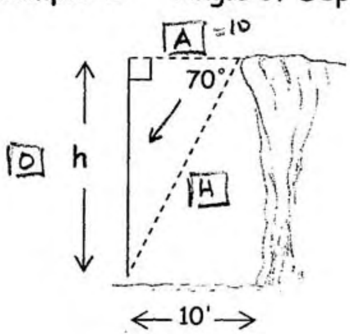
$\tan 59° = \frac{h}{45}$

$\frac{1.6643}{1} = \frac{h}{45}$

$45(1.6643) = h$

$\boxed{h = 75' \text{ approx.}}$

Example 2 Angle of Depression: Find the height of the cliff.



TOA $\quad \tan = \frac{O}{A}$

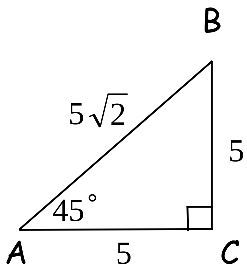
$\tan 70° = \frac{h}{10}$

$\frac{2.7475}{1} = \frac{h}{10}$

$10(2.7475) = h$

$\boxed{27' = h \text{ approx.}}$

1.) Find the sine, cosine, and tangent of the acute angles of the triangle. Express each value as a fraction and a decimal rounded to 4 places.



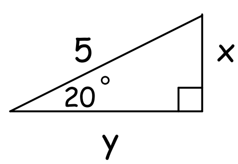
	fraction	decimal			fraction	decimal
sin A				sin B		
cos A				cos B		
tan A				tan B		

Use the table of trigonometric Ratios to find the given value. Decimal should be to 4 places.

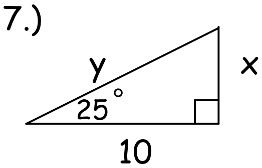
2.) $\sin 10^\circ =$ 3.) $\cos 38^\circ =$ 4.) $\tan 44^\circ =$ 5.) $\sin 74^\circ =$

Find the value of each variable.

6.)



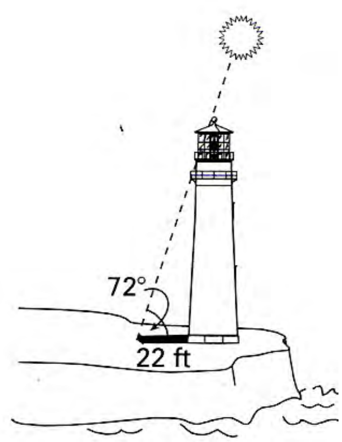
7.)



Use the figure of the lighthouse for #8-9

8.) At 2 P.M. the shadow of a lighthouse is 22 ft. long and the angle of elevation is 72°. Find the height of the lighthouse.

9.) At 4 P.M. the angle of elevation of the sun is 40°. Find the length of the shadow cast by the lighthouse.

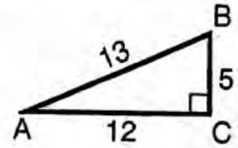


(Adapted from Larson, 2001b, p. 74)

For each exercise, select the correct ratio from the four choices given.
Write the letter of the correct choice in the box that contains the number of that exercise.

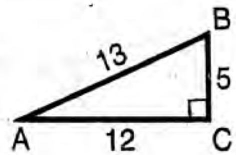
1. sin A =
2. cos A =
3. tan A =

(E) $\frac{12}{13}$ (A) $\frac{5}{13}$ (F) $\frac{5}{12}$ (V) $\frac{13}{5}$



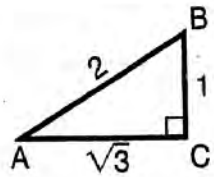
4. sin B =
5. cos B =
6. tan B =

(N) $\frac{13}{5}$ (T) $\frac{5}{13}$ (R) $\frac{12}{13}$ (O) $\frac{12}{5}$



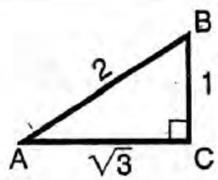
7. sin A =
8. cos A =
9. tan A =

(A) $\frac{\sqrt{3}}{2}$ (R) $\frac{1}{2}$ (S) 2 (H) $\frac{1}{\sqrt{3}}$



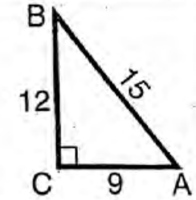
10. sin B =
11. cos B =
12. tan B =

(A) $\sqrt{3}$ (N) $\frac{1}{2}$ (O) $\frac{\sqrt{3}}{2}$ (P) $\frac{1}{\sqrt{3}}$



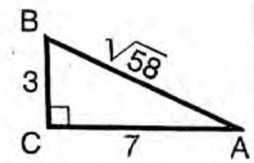
13. sin A =
14. cos A =
15. tan A =

(U) $\frac{5}{3}$ (S) $\frac{3}{5}$ (L) $\frac{4}{3}$ (E) $\frac{4}{5}$



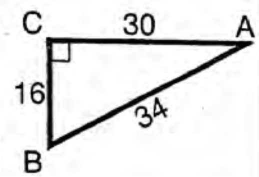
16. sin B =
17. cos B =
18. tan B =

(D) $\frac{3}{\sqrt{58}}$ (B) $\frac{3}{7}$ (O) $\frac{7}{\sqrt{58}}$ (A) $\frac{7}{3}$



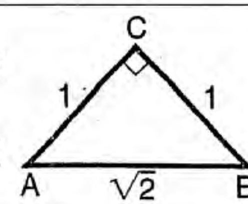
19. sin A =
20. cos A =
21. tan A =

(N) $\frac{15}{17}$ (O) $\frac{8}{17}$ (R) $\frac{17}{8}$ (C) $\frac{8}{15}$



22. sin B =
23. cos B =
24. tan B =

(W) $\frac{1}{\sqrt{2}}$ (W) $\frac{1}{\sqrt{2}}$ (A) 1 (L) $\sqrt{2}$



I'm the part of the bird that's not in the sky. I can swim in the ocean and yet remain dry.

1	14	9	18	17	6	22

You throw away the outside and cook the inside. Then you eat the outside and throw away the inside. What is it ?

8	20	2	12	4	19	3	21	10	7	11

What gets wetter the more it dries ?

24	5	16	23	13	15

(Adapted from Marcy, 1996, p. 227)

Lesson Plan #4: Trigonometric Ratios Teacher Notes

Warm-up

1.) Substitute the values for a, b, and c into the following ratios then write the fraction as a decimal.

a = 3

b = 4 $\frac{a}{c} =$ $\frac{b}{c} =$ $\frac{a}{b} =$ $\frac{b}{c} =$

c = 5

2.) Rationalize the following.

$\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{3}}$

Lesson Plan #4: Trigonometric Ratios Chart

TRIGONOMETRIC RATIOS

Angle	sin	cos	tan	Angle	sin	cos	tan
0°	0.0000	1.0000	0.0000	45°	0.7071	0.7071	1.0000
1°	0.0175	0.9998	0.0175	46°	0.7193	0.6947	1.0355
2°	0.0349	0.9994	0.0349	47°	0.7314	0.6820	1.0724
3°	0.0523	0.9986	0.0524	48°	0.7431	0.6691	1.1106
4°	0.0698	0.9976	0.0699	49°	0.7547	0.6561	1.1504
5°	0.0872	0.9962	0.0875	50°	0.7660	0.6428	1.1918
6°	0.1045	0.9945	0.1051	51°	0.7771	0.6293	1.2349
7°	0.1219	0.9925	0.1228	52°	0.7880	0.6157	1.2799
8°	0.1392	0.9903	0.1405	53°	0.7986	0.6018	1.3270
9°	0.1564	0.9877	0.1584	54°	0.8090	0.5878	1.3764
10°	0.1736	0.9848	0.1763	55°	0.8192	0.5736	1.4281
11°	0.1908	0.9816	0.1944	56°	0.8290	0.5592	1.4826
12°	0.2079	0.9781	0.2126	57°	0.8387	0.5446	1.5399
13°	0.2250	0.9744	0.2309	58°	0.8480	0.5299	1.6003
14°	0.2419	0.9703	0.2493	59°	0.8572	0.5150	1.6643
15°	0.2588	0.9659	0.2679	60°	0.8660	0.5000	1.7321
16°	0.2756	0.9613	0.2867	61°	0.8746	0.4848	1.8040
17°	0.2924	0.9563	0.3057	62°	0.8829	0.4695	1.8807
18°	0.3090	0.9511	0.3249	63°	0.8910	0.4540	1.9626
19°	0.3256	0.9455	0.3443	64°	0.8988	0.4384	2.0503
20°	0.3420	0.9397	0.3640	65°	0.9063	0.4226	2.1445
21°	0.3584	0.9336	0.3839	66°	0.9135	0.4067	2.2460
22°	0.3746	0.9272	0.4040	67°	0.9205	0.3907	2.3559
23°	0.3907	0.9205	0.4245	68°	0.9272	0.3746	2.4751
24°	0.4067	0.9135	0.4452	69°	0.9336	0.3584	2.6051
25°	0.4226	0.9063	0.4663	70°	0.9397	0.3420	2.7475
26°	0.4384	0.8988	0.4877	71°	0.9455	0.3256	2.9042
27°	0.4540	0.8910	0.5095	72°	0.9511	0.3090	3.0777
28°	0.4695	0.8829	0.5317	73°	0.9563	0.2924	3.2709
29°	0.4848	0.8746	0.5543	74°	0.9613	0.2756	3.4874
30°	0.5000	0.8660	0.5774	75°	0.9659	0.2588	3.7321
31°	0.5150	0.8572	0.6009	76°	0.9703	0.2419	4.0108
32°	0.5299	0.8480	0.6249	77°	0.9744	0.2250	4.3315
33°	0.5446	0.8387	0.6494	78°	0.9781	0.2079	4.7046
34°	0.5592	0.8290	0.6745	79°	0.9816	0.1908	5.1446
35°	0.5736	0.8192	0.7002	80°	0.9848	0.1736	5.6713
36°	0.5878	0.8090	0.7265	81°	0.9877	0.1564	6.3138
37°	0.6018	0.7986	0.7536	82°	0.9903	0.1392	7.1154
38°	0.6157	0.7880	0.7813	83°	0.9925	0.1219	8.1443
39°	0.6293	0.7771	0.8098	84°	0.9945	0.1045	9.5144
40°	0.6428	0.7660	0.8391	85°	0.9962	0.0872	11.4301
41°	0.6561	0.7547	0.8693	86°	0.9976	0.0698	14.3007
42°	0.6691	0.7431	0.9004	87°	0.9986	0.0523	19.0811
43°	0.6820	0.7314	0.9325	88°	0.9994	0.0349	28.6363
44°	0.6947	0.7193	0.9657	89°	0.9998	0.0175	57.2900

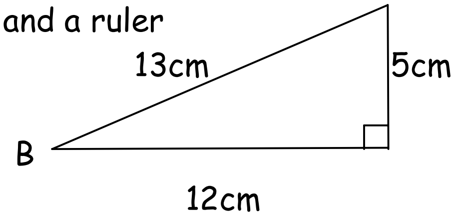
(Trigonometric Ratios, 2007, p. 5)

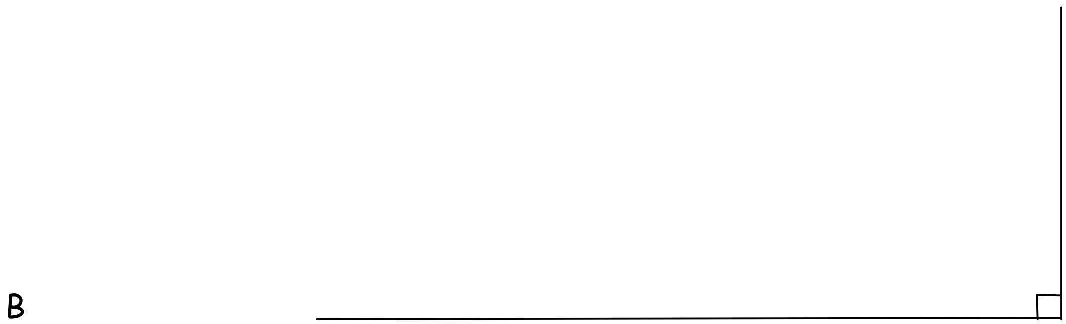
APPENDIX J

Solving Right Triangles

Lesson Plan #5 Activity: Solving Right Triangles Activity

1.) Using the right angle provided below and a ruler to draw a triangle with side lengths of 5 cm., 12 cm., and 13 cm.
(As shown in the diagram to the right).
Measure as accurately as possible.

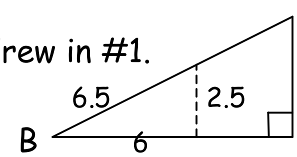




a.) Write the following trigonometric ratios as a fraction and as a decimal rounded to 4 places. sin B = cos B = tan B =	b.) Use the Table of Trigonometric Ratios to determine the measure of angle B. Look for the decimal under the appropriate column then read across to determine the angle. When the sin B = the $m\angle B =$ When the cos B = the $m\angle B =$ When the tan B = the $m\angle B =$

c.) Measure angle B with a protractor. Protractor measurement: $m\angle B =$

2.) Draw a similar triangle inside the triangle you drew in #1.
Use side lengths 2.5 cm., 6 cm., and 6.5 cm.



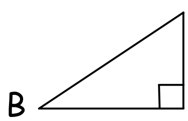
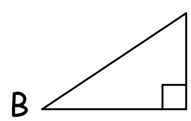
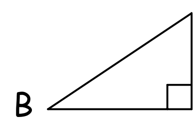
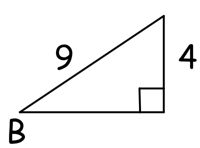
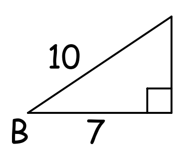
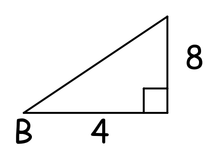
Then write the following trigonometric ratios as a fraction and as a decimal rounded to 4 places. sin B = cos B = tan B =

Are the ratios the same? Explain why.

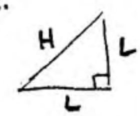
Lesson Plan #5: Solving Right Triangles Notes

Parts of a Right Triangle:	Solve a Right Triangle:	Needed to Solve a Right Triangle:

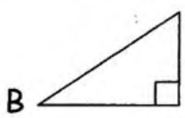
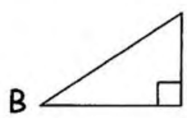
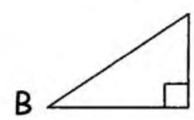
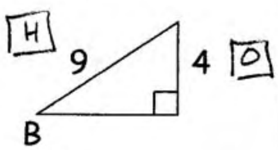
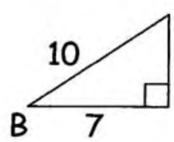
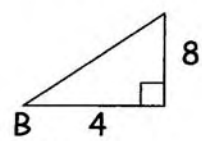
Finding the Measure of Angles in Right Triangles

1.) $\sin B = \frac{O}{H}$  	3.) $\cos B = \frac{A}{H}$  	5.) $\tan B = \frac{O}{A}$ 
2.) Find $m\angle B$  	4.) Find $m\angle B$  	6.) Find $m\angle B$ 

Key Lesson Plan #5: Solving Right Triangles Notes

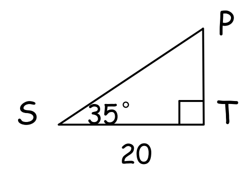
Parts of a Right Triangle:	Solve a Right Triangle:	Needed to Solve a Right Triangle:
6 parts  3 sides: 1 hypotenuse 2 legs 3 angles: 1 right angle 2 acute angles	this means determine the measure of all 6 parts	either 2 side lengths or 1 side length & 1 acute angle Use: SOH-CAH-ToA Pythagorean Th. Sum of angles = 180

Finding the Measure of Angles in Right Triangles

1.) $\sin B = \frac{O}{H}$  read: the inverse sine of $\sin^{-1}\left(\frac{O}{H}\right) = m\angle B$ the angle whose sin is this number	3.) $\cos B = \frac{A}{H}$  $\cos^{-1}\left(\frac{A}{H}\right) = m\angle B$ The angle whose cosine is this number	5.) $\tan B = \frac{O}{A}$  $\tan^{-1}\left(\frac{O}{A}\right) = m\angle B$ The angle whose tangent is this number
2.) Find $m\angle B$  $\sin^{-1}\left(\frac{4}{9}\right) = m\angle B$ $\sin^{-1}(.4444) = m\angle B$ on trig chart between $26°$ and $27°$ $m\angle B \approx 26.5°$ Calculator $m\angle B = 26.4°$	4.) Find $m\angle B$  $\cos^{-1}\left(\frac{7}{10}\right) = m\angle B$ $\cos^{-1}(.7000) = m\angle B$ on trig. chart between $45°$ & $46°$ $m\angle B \approx 45.5°$ **Calculator $m\angle B = 45.6$**	6.) Find $m\angle B$  $\tan^{-1}\left(\frac{8}{4}\right) = m\angle B$ $\tan^{-1}(2) = m\angle B$ on trig chart between $63°$ & $64°$ $m\angle B \approx 63.5°$ Calculator $m\angle B = 63.4°$

Solving Right Triangles:

7.) <u>Given: 1 acute angle & 1 side</u>



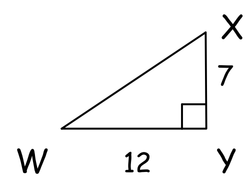
$m\angle S = 35^\circ$

ST = 20

Find:

a.) $m\angle P$	b.) SP	c.) PT

8.) <u>Given: 2 sides</u>



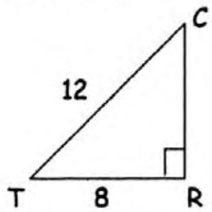
WY = 12

XY = 7

Find:

a.) WX	b.) $m\angle W$	c.) $m\angle X$

Solve the triangle. Use either the pythagorean theorem or trigonometric ratios to find the indicated measurements.

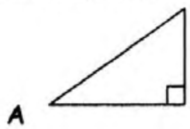


1.) CR =

2.) m∠T =

3.) m∠C =

∠A is an acute angle. Use the table of Trig. Ratios to approximate the measure of ∠A.



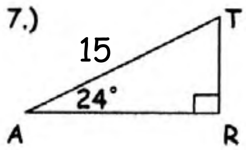
4.) sin A = .0800

5.) cos A = .9400

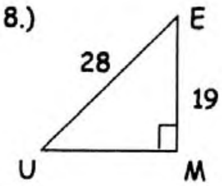
6.) tan A = .8700

Solve the triangles. Use either the pythagorean theorem or trigonometric ratios to find the measurements of the missing sides and angles. Round decimals to the nearest tenth.

7.)

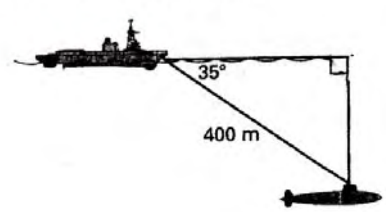


8.)



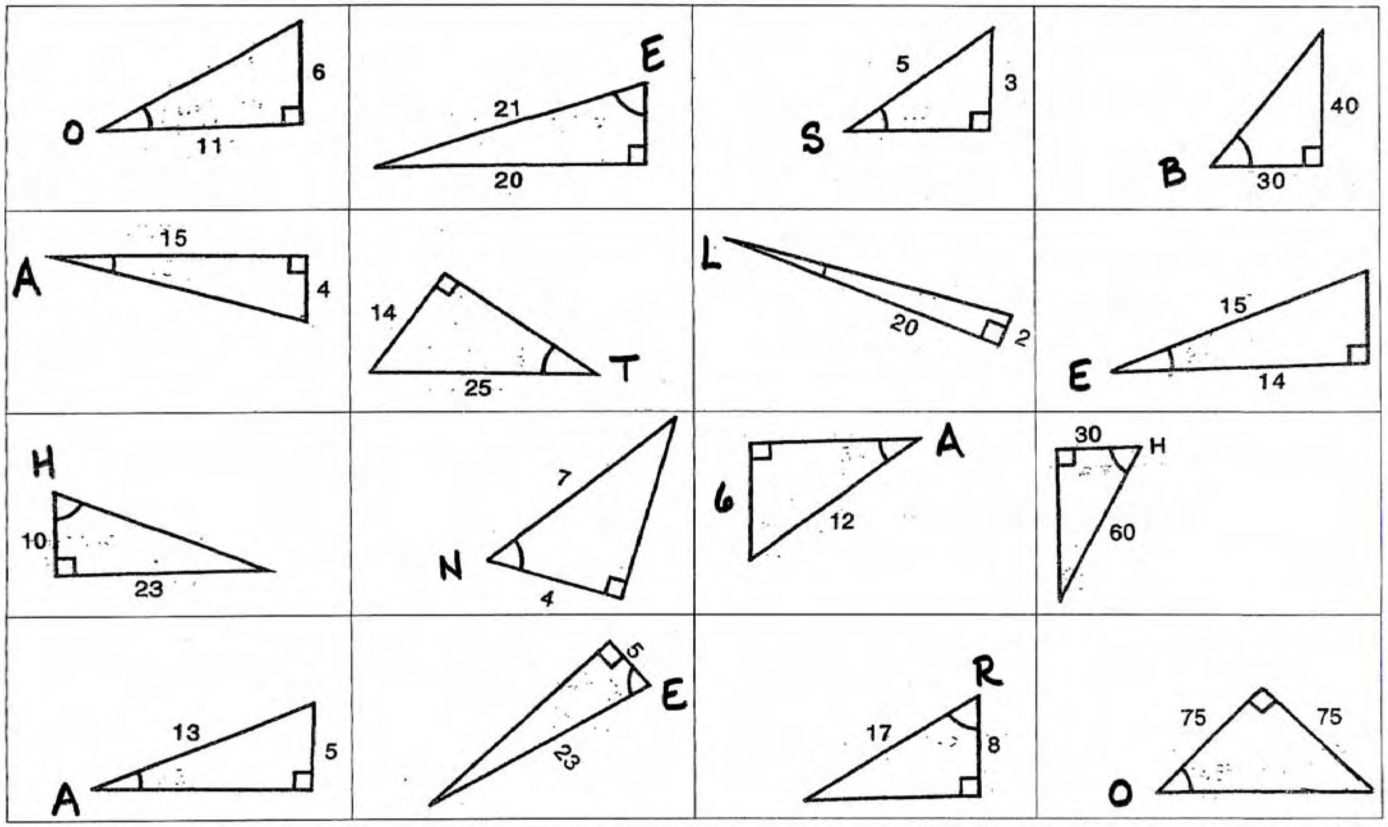
9.) A ramp was built by a loading dock. The height of the loading platform is 4 ft. Determine the length of the ramp if it makes a 32° angle with the ground.

10.) A sonar operator on a ship detects a submarine at a distance of 400 meters and an angle of depression of 35°. How deep is the submarine ?



(Adapted from Larson, 2001b, p. 90)

For each triangle, find the measure of the lettered angle to the nearest degree (use the trigonometry ratio chart). Write the letter in the box that contains the measure of the angle.



I can run but not walk. Whenever I go, thought follows close behind.

23	55	29	37	21

I'm light as a feather, yet the strongest man can't hold me for more than a minute.

53	62	72	30	34	60

I am weightless, but you can see me. Put me in a bucket and I'll make it lighter.

15	67	45	6	77

APPENDIX K

Clinometer Activity

Lesson Plan #6: Clinometer Activity

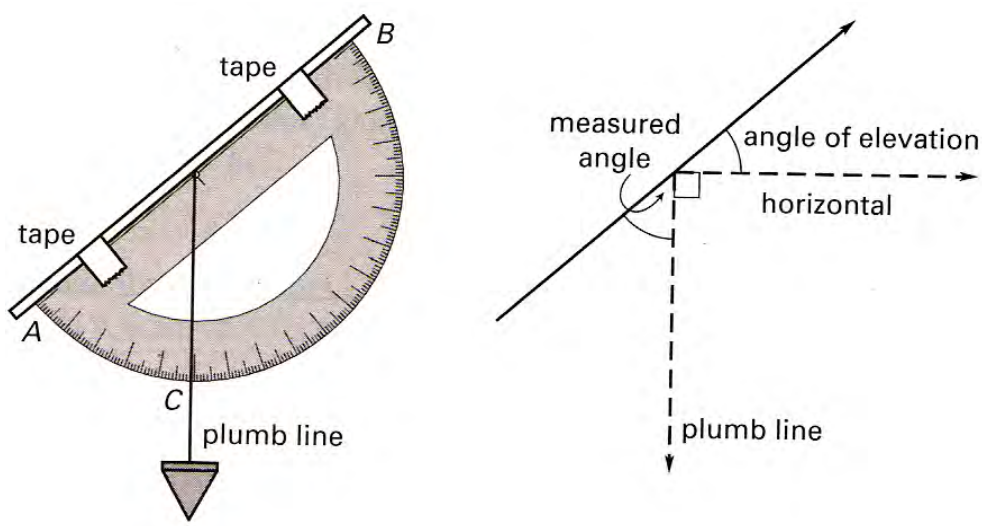
Background Information: Trigonometric ratios can be used to determine the height of difficult to measure objects. An instrument called a clinometer is used by foresters to measure the angle of elevation from the ground to the top of a tree. Using the angle of elevation, the distance to the tree, and trigonometric ratios the height of the tree can be determined. This technique can also be used to find the height of other tall objects.

Part I : Constructing the Clinometer

Materials needed: Protractor, drinking straw, string, tape, 1" prong paper fastener or large paper clip, measuring tape or meter stick, trigonometric ratio chart, calculator.

Directions: Cut a 16" length of string. Tape the string to the middle of the flat side of the protractor (tape the string on one side then hang it over the other side so it can swing freely). Cut a 5" piece of the straw. Tape the straw to the top of the flat side of the protractor (the string should still swing freely). Attach the paper fastener to the end of the string.

The straw will serve as the viewing tube. The string (plumb line) crosses the angle measurements on the protractor at point C, as shown in the diagram below, forming an acute angle ($\angle AOC$). This angle is the compliment of the angle of elevation.



(Adapted from Larson, 2001b, p. 78)

Part II: Using the Clinometer

1. Measure you eye height. Eye height ______________

2. Locate a tall object that would be difficult to measure directly. Tall object _______________

3. Use the clinometer to measure the observer's viewing angle to the top of the object. Angle of elevation __________

4. Measure the distance from the observer to the base of the object. Distance _________________

5. Draw a diagram below and label it with the information that you collected.

Analyzing the Results:

6. Label the sides of right triangle formed in your diagram with H, O, A. Which trigonometric function should be used? _______________

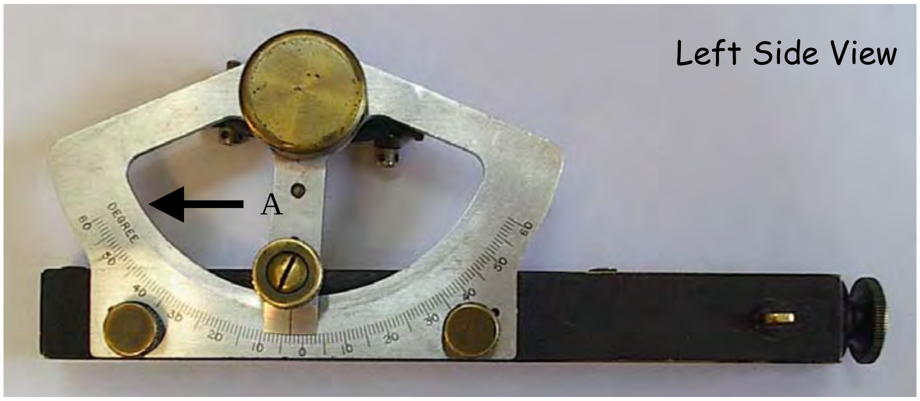
7. Write an equation then determine the height of the object. Show your work below.

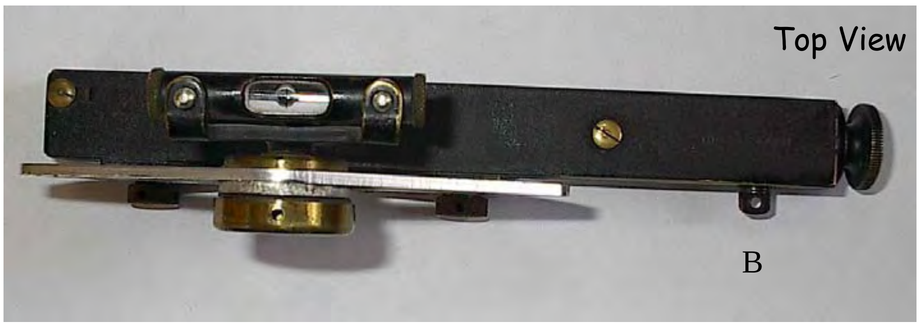
Height ______________

8. Describe or draw a diagram to show another way to find the height of the tall object if you had not measured the observer's eye height (or anything else on the observer), but had measured some other height or angle.

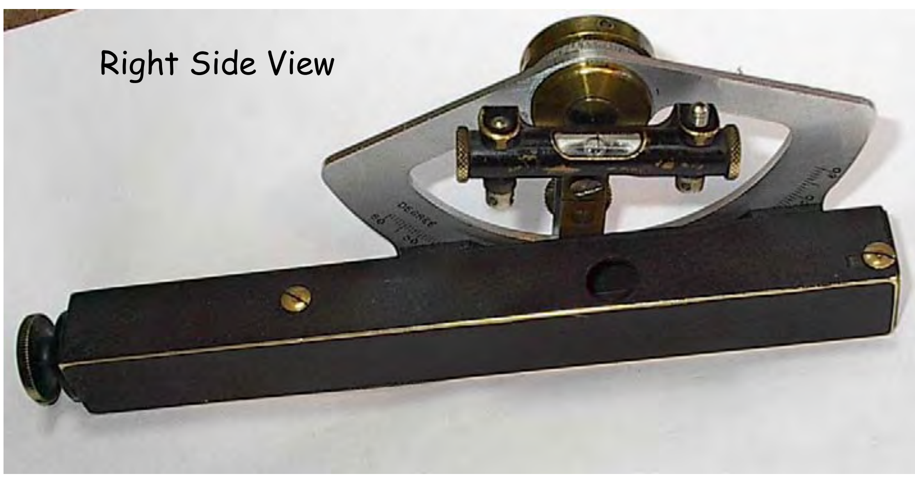
(Adapted from Larson, 2001b, p. 78)

Photographs of a Clinometer.





Plumb bob



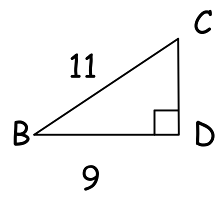
To read the angle of elevation with a clinometer the observer looks through the clinometer and sights the top of the tree. The view is split so the viewer can see both the tree and the level bubble. The viewer then moves the arm (point A in top view) until the bubble is centered in the viewing screen. Next the viewer looks at the scale and reads the angle.

A plumb bob can be attached to the side (point B side view) to determine the exact point on the ground from which the viewer is reading the angle of elevation.

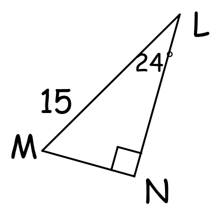
Lesson Plan #6: Clinometer Activity & Review Teacher Notes

Warm-up

1.) Solve the Triangles



$m\angle B =$ $m\angle C =$ CD =



$m\angle M =$ LN = MN =

Clinometer Activity

Prior to the activity the teacher should choose an area outdoors with several tall objects such as a flag pole, trees, and light posts. Prior to the lesson the teacher should draw a plan of the location of the objects on the blackboard, or the overhead, and instruct students to choose one of the designated objects. When the activity has been completed the students will label the object with the height that they calculated.

If the teacher has more than one geometry class, he or she may also record the heights calculated by all of the classes to show the students how they compared.

If possible the teacher should borrow a clinometer from someone who works for the Bureau of Land Management, the National Park Service or a private company. Students can try using the real clinometer during the activity or afterwards.

Extension: Students use the clinometer at home to determine the height of a tall object. They should complete the questions on the back of Activity #6. This assignment can be used as homework or extra credit.